**2**

# OPERATIONS AND PROPERTIES

Jesse is fascinated by number relationships and often tries to find special mathematical properties of the five-digit number displayed on the odometer of his car. Today Jesse noticed that the number on the odometer was a palindrome and an even number divisible by 11, with 2 as three of the digits. What was the five-digit reading? (Note: A palindrome is a number, word, or phrase that is the same read left to right as read right to left, such as 57375 or Hannah.)

In this chapter you will review basic operations of arithmetic and their properties. You will also study operations on sets.

## 2-1 ORDER OF OPERATIONS

### The Four Basic Operations in Arithmetic

Bicycles have two wheels. Bipeds walk on two feet. Biceps are muscles that have two points of origin. Bilingual people can speak two languages. What do these *bi*-words have in common with the following examples?

$$6.3 + 0.9 = 7.2 \qquad\qquad 21.4 \times 3 = 64.2$$

$$11\tfrac{3}{7} - 2\tfrac{1}{7} = 9\tfrac{2}{7} \qquad\qquad 9 \div 2 = 4\tfrac{1}{2}$$

The prefix *bi-* means "two." In each example above, an operation or rule was followed to replace *two* rational numbers with a *single* rational number. These familiar operations of addition, subtraction, multiplication, and division are called **binary operations**. Each of these operations can be performed with any pair of rational numbers, except that division by zero is meaningless and is not allowed.

In every binary operation, two elements from a set are replaced by exactly one element from the same set. There are some important concepts to remember when working with binary operations:

1. A *set* must be identified, such as the set of whole numbers or the set of rational numbers. When no set is identified, use the set of all real numbers.

2. The *rule* for the binary operation must be clear, such as the rules you know for addition, subtraction, multiplication, and division.

3. The order of the elements is important. Later in this chapter, we will use the notation $(a, b)$ to indicate an *ordered pair* in which $a$ is the first element and $b$ is the second element. For now, be aware that answers may be different depending on which element is first and which is second. Consider subtraction. If 8 is the first element and 5 is the second element, then: $8 - 5 = 3$. But if 5 is the first element and 8 is the second element, then: $5 - 8 = -3$

4. Every problem using a binary operation must have an *answer*, and there must be *only one* answer. We say that each answer is *unique*, meaning there is *one and only one* answer.

**DEFINITION**

A **binary operation** in a set assigns to every ordered pair of elements from the set a unique answer from the set.

Note that, even when we find the sum of three or more numbers, we still add only two numbers at a time, indicating the binary operation:

$$4 + 9 + 7 = (4 + 9) + 7 = 13 + 7 = 20$$

# Factors

When two or more numbers are multiplied to give a certain product, each number is called a **factor** of the product. For example:

- Since $1 \times 16 = 16$, then 1 and 16 are factors of 16.
- Since $2 \times 8 = 16$, then 2 and 8 are factors of 16.
- Since $4 \times 4 = 16$, then 4 is a factor of 16.
- The numbers 1, 2, 4, 8, and 16 are all factors of 16.

# Prime Numbers

A **prime number** is a whole number greater than 1 that has no whole number factors other than itself and 1. The first seven prime numbers are 2, 3, 5, 7, 11, 13, 17. Whole numbers greater than 1 that are not prime are called **composite numbers**. Composite numbers have three or more whole number factors. Some examples of composite numbers are 4, 6, 8, 9, 10.

# Bases, Exponents, Powers

When the same number appears as a factor many times, we can rewrite the expression using *exponents*. For example, the exponent 2 indicates that the factor appears twice. In the following examples, the repeated factor is called a *base*.

$4 \times 4 = 16$ can be written as $4^2 = 16$.

$4^2$ is read as "4 squared," or "4 raised to the second power," or "the second power of 4."

$$\text{Base} \longrightarrow 4^2 = 16 \longleftarrow \text{Power} \qquad \text{Exponent}$$

The exponent 3 indicates that a factor is used three times.

$4 \times 4 \times 4 = 64$ can be written as $4^3 = 64$.

$4^3$ is read as "4 cubed," or "4 raised to the third power," or "the third power of 4."

$$\text{Base} \longrightarrow 4^3 = 64 \longleftarrow \text{Power} \qquad \text{Exponent}$$

The examples shown above lead to the following definitions:

**DEFINITION**

A **base** is a number that is used as a factor in the product.

An **exponent** is a number that tells how many times the base is to be used as a factor. The exponent is written, in a smaller size, to the upper right of the base.

A **power** is a number that is a product in which all of its factors are equal.

A number raised to the first power is equal to the number itself, as in $6^1 = 6$. Also, when no exponent is shown, the exponent is 1, as in $9 = 9^1$.

**EXAMPLE 1**

Compute the value of $4^5$.

*Solution*   $4 \times 4 \times 4 \times 4 \times 4 = 1,024$

*Calculator Solution*   Use the exponent key, $\boxed{\wedge}$, on a calculator.

ENTER: 4 $\boxed{\wedge}$ 5 $\boxed{\textbf{ENTER}}$

DISPLAY:

```
4^5
              1024
```

*Answer*   1,024

**EXAMPLE 2**

Find $\left(\frac{2}{3}\right)^3$ **a.** as an exact value **b.** as a rational approximation.

*Solution*   **a.** The exact value is a fraction:

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$$

**b.** Use a calculator.
ENTER: $\boxed{(}$ 2 $\boxed{\div}$ 3 $\boxed{)}$ $\boxed{\wedge}$ 3 $\boxed{\textbf{ENTER}}$

DISPLAY:

```
(2/3)^3
           .2962962963
```

**Note:** The exact value is a rational number that can also be written as the repeating decimal $0.\overline{296}$.

*Answers*   **a.** $\frac{8}{27}$   **b.** 0.2962962963

## Computations With More Than One Operation

When a numerical expression involves two or more different operations, we need to agree on the order in which they are performed. Consider this example:

$$11 - 3 \times 2$$

Suppose that one person multiplied first.

$$11 - 3 \times 2 = 11 - 6$$
$$= 5$$

Suppose another person subtracted first.

$$11 - 3 \times 2 = 8 \times 2$$
$$= 16$$

Who is correct?

In order that there will be one and only one correct answer to problems like this, mathematicians have agreed to follow this **order of operations**:

**1.** Simplify powers (terms with exponents).

**2.** Multiply and divide, from left to right.

**3.** Add and subtract, from left to right.

Therefore, we multiply before we subtract, and $11 - 3 \times 2 = 11 - 6 = 5$ is correct.

A different problem involving powers is solved in this way:

**1.** Simplify powers:          $5 \times 2^3 + 3 = 5 \times 8 + 3$

**2.** Multiply and divide:          $= 40 + 3$

**3.** Add and subtract:          $= 43$

## Expressions with Grouping Symbols

In mathematics, **parentheses ( )** act as grouping symbols, giving different meanings to expressions. For example, $(4 \times 6) + 7$ means "add 7 to the product of 4 and 6," while $4 \times (6 + 7)$ means "multiply the sum of 6 and 7 by 4."

When simplifying any numerical expression, always perform the operations within parentheses first.

$$(4 \times 6) + 7 = 24 + 7 = 31 \qquad 4 \times (6 + 7) = 4 \times 13 = 52$$

Besides parentheses, other symbols are used to indicate grouping, such as **brackets [ ]**. The expressions $2(5 + 9)$ and $2[5 + 9]$ have the same meaning: 2 is multiplied by the sum of 5 and 9. A *bar*, or *fraction line*, also acts as a symbol of grouping, telling us to perform the operations in the numerator and/or denominator first.

$$\frac{20 - 8}{3} = \frac{12}{3} = 4 \qquad\qquad \frac{6}{3 + 1} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$$

However, when entering expressions such as these into a calculator, the line of the fraction is usually entered as a division and a numerator or denominator that involves an operation must be enclosed in parentheses.

ENTER: ( ) 20 ( − ) 8 ( ) ) ( ÷ ) 3 **ENTER**

DISPLAY:

```
(20−8)/3
                    4
```

ENTER: 6 ( ÷ ) ( ( ) 3 ( ÷ ) 1 ( ) ) **ENTER**

DISPLAY:

```
6/(3+1)
                    1.5
```

When there are *two or more* grouping symbols in an expression, we perform the operations on the numbers in the *innermost symbol first*. For example:

$$5 + 2[6 + (3 − 1)^3]$$
$$= 5 + 2[6 + 2^3]$$
$$= 5 + 2[6 + 8]$$
$$= 5 + 2[14]$$
$$= 5 + 28$$
$$= 33$$

### Procedure

**To simplify a numerical expression, follow the correct order of operations:**

1. Simplify any numerical expressions within parentheses or within other grouping symbols, starting with the innermost.

2. Simplify any powers.

3. Do all multiplications and divisions in order from left to right.

4. Do all additions and subtractions in order from left to right.

### EXAMPLE 3

Simplify the numerical expression $80 − 4(7 − 5)$.

***Solution*** Remember that, in the given expression, $4(7 - 5)$ means 4 *times* the value in the parentheses.

*How to Proceed*

| | |
|---|---|
| (1) Write the expression: | $80 - 4(7 - 5)$ |
| (2) Simplify the value within the parentheses: | $= 80 - 4(2)$ |
| (3) Multiply: | $= 80 - 8$ |
| (4) Subtract: | $= 72$ |

***Calculator Solution*** ENTER: 80 $-$ 4 ( 7 $-$ 5 ) ENTER

DISPLAY:

```
80-4(7-5)
            72
```

*Answer* 72

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## EXERCISES

### Writing About Mathematics

**1.** Explain why 2 is the only even prime.

**2.** Delia knows that every number except 2 that ends in a multiple of 2 is composite. Therefore, she concludes that every number except 3 that ends in a multiple of 3 is composite. Is Delia correct? Explain how you know.

### Developing Skills

In 3–10, state the meaning of each expression in part **a** and in part **b**, and simplify the expression in each part.

**3. a.** $20 + (6 + 1)$  **b.** $20 + 6 + 1$   **4. a.** $18 - (4 + 3)$  **b.** $18 - 4 + 3$

**5. a.** $12 - (3 - 0.5)$  **b.** $12 - 3 - 0.5$   **6. a.** $15 \times (2 + 1)$  **b.** $15 \times 2 + 1$

**7. a.** $(12 + 8) \div 4$  **b.** $12 + 8 \div 4$   **8. a.** $48 \div (8 - 4)$  **b.** $48 \div 8 - 4$

**9. a.** $7 + 5^2$  **b.** $(7 + 5)^2$   **10. a.** $4 \times 3^2$  **b.** $(4 \times 3)^2$

**11.** Noella said that since the line of a fraction indicates division, $\frac{10 \times 15}{5 \times 3}$ is the same as $10 \times 15 \div 5 \times 3$. Do you agree with Noella? Explain why or why not.

In 12–15: **a.** Find, in each case, the value of the three given powers. **b.** Name, in each case, the expression that has the greatest value.

**12.** $5^2, 5^3, 5^4$        **13.** $(0.5)^2, (0.5)^3, (0.5)^4$      **14.** $(0.5)^2, (0.6)^2, (0.7)^2$      **15.** $(1.1)^2, (1.2)^2, (1.3)^2$

In 16–23: **a.** List all of the whole numbers that are factors of each of the given numbers. **b.** Is the number prime, composite, or neither?

| | | | |
|---|---|---|---|
| **16.** 82 | **17.** 101 | **18.** 71 | **19.** 15 |
| **20.** 1 | **21.** 808 | **22.** 67 | **23.** 397 |

## Applying Skills

In 24–28, write a numerical expression for each of the following and find its value to answer the question.

**24.** What is the cost of two chocolate chip and three peanut butter cookies if each cookie costs 28 cents?

**25.** What is the cost of two chocolate chip cookies that cost 30 cents each and three peanut butter cookies that cost 25 cents each?

**26.** How many miles did Ms. McCarthy travel if she drove 30 miles per hour for $\frac{3}{4}$ hour and 55 miles per hour for $1\frac{1}{2}$ hours?

**27.** What is the cost of two pens at $0.38 each and three notebooks at $0.69 each?

**28.** What is the cost of five pens at $0.29 each and three notebooks at $0.75 each if ordered from a mail order company that adds $1.75 in postage and handling charges?

In 29–30, use a calculator to find each answer.

**29.** The value of $1 invested at 6% for 20 years is equal to $(1.06)^{20}$. Find, to the nearest cent, the value of this investment after 20 years.

**30.** The value of $1 invested at 8% for $n$ years is equal to $(1.08)^n$. How many years will be required for $1 invested at 8% to double in value? (*Hint:* Guess at values of $n$ to find the value for which $(1.08)^n$ is closest to 2.00.)

**31.** In each box insert an operational symbol $+, -, \times, \div$, and then insert parentheses if needed to make each of the following statements true.

**a.** $3 \ \square \ 2 \ \square \ 1 = 4$        **b.** $1 \ \square \ 3 \ \square \ 1 = 4$        **c.** $1 \ \square \ 2 \ \square \ 3 \ \square \ 4 = 5$

**d.** $4 \ \square \ 3 \ \square \ 2 \ \square \ 1 = 5$      **e.** $6 \ \square \ 6 \ \square \ 6 \ \square \ 6 = 5$      **f.** $6 \ \square \ 6 \ \square \ 6 \ \square \ 6 = 6$

## 2-2 PROPERTIES OF OPERATIONS

When numbers behave in a certain way for an operation, we describe this behavior as a **property**. You are familiar with these operations from your study of arithmetic. As we examine the properties of operations, no proofs are given, but the examples will help you to see that these properties make sense and to identify the sets of numbers for which they are true.

## The Property of Closure

A set is said to be **closed** under a binary operation when every pair of elements from the set, under the given operation, yields an element from that set.

**1.** Add any two numbers.

$23 + 11 = 34$     The sum of two whole numbers is a whole number.

$7.8 + 4.8 = 12.6$  The sum of two rational numbers is a rational number.

$\pi + (-\pi) = 0$     The sum of $\pi$ and its opposite, $-\pi$, two irrational numbers, is 0, a rational number.

Even though the sum of two irrational numbers is usually an irrational number, the set of irrational numbers is not closed under addition. There are some pairs of irrational numbers whose sum is not an irrational number. However, $\pi$, $-\pi$, 0, and each of the other numbers used in these examples are real numbers and the sum of two real numbers is a real number.

▶ **The sets of whole numbers, rational numbers, and real numbers are each closed under addition.**

**2.** Multiply any two numbers.

$(2)(4) = 8$                The product of two whole numbers is a whole number.

$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$                The product of two rational numbers is a rational number.

$\sqrt{2} \times \sqrt{2} = \sqrt{4} = 2$  The product of $\sqrt{2}$ and $\sqrt{2}$, two irrational numbers, is 2, a rational number.

Though the product of two irrational numbers is usually an irrational number, there are some pairs of irrational numbers whose product is not an irrational number. The set of irrational numbers is not closed under multiplication. However, $\sqrt{2}$, 2, and each of the other numbers used in these examples are real numbers and the product of two real numbers is a real number.

▶ **The sets of whole numbers, rational numbers, and real numbers are each closed under multiplication.**

**3.** Subtract any two numbers.

$7 - 12 = -5$    The difference of two whole numbers is not a whole number, but these whole numbers are also integers and the difference between two integers is an integer.

$12.7 - 8.2 = 4.5$    The difference of two rational numbers is a rational number.

$\sqrt{3} - \sqrt{3} = 0$    The difference of $\sqrt{3}$ and $\sqrt{3}$, two irrational numbers, is 0, a rational number.

Even though the difference of two irrational numbers is usually an irrational number, there are some pairs of irrational numbers whose difference is not an irrational number. The set of irrational numbers is not closed under subtraction. However, $\sqrt{3}, 0$, and each of the other numbers used in these examples are real numbers and the difference of two real numbers is a real number.

▶ **The sets of integers, rational numbers, and real numbers are each closed under subtraction.**

**4.** Divide any two numbers by a nonzero number. (Remember that division by 0 is not allowed.)

$9 \div 2 = 4.5$    The quotient of two whole numbers or two integers is not always a whole number or an integer.

$\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$    The quotient of two rational numbers is a rational number.

$\sqrt{5} \div \sqrt{5} = 1$    The quotient of $\sqrt{5}$ and $\sqrt{5}$, two irrational numbers, is 1, a rational number.

Though the quotient of two irrational numbers is usually an irrational number, there are some pairs of irrational numbers whose quotient is not an irrational number. The set of irrational numbers is not closed under division. However, $\sqrt{5}, 1$, and each of the other numbers used in these examples are real numbers and the quotient of two nonzero real numbers is a nonzero real number.

▶ **The sets of nonzero rational numbers, and nonzero real numbers are each closed under division.**

Later in this book, we will study operations with signed numbers and operations with irrational numbers in greater detail. For now, we will simply make these observations:

▶ **The set of whole numbers is closed under the operations of addition and multiplication.**

► **The set of integers is closed under the operations of addition, subtraction, and multiplication.**

► **The set of rational numbers is closed under the operations of addition, subtraction, and multiplication, and the set of nonzero rational numbers is closed under division.**

► **The set of real numbers is closed under the operations of addition, subtraction, and multiplication, and the set of nonzero real numbers is closed under division.**

## Commutative Property of Addition

When we add rational numbers, we assume that we can change the *order* in which two numbers are added without changing the sum.

For example, $4 + 5 = 5 + 4$ and $\frac{1}{2} + \frac{1}{4} = \frac{1}{4} + \frac{1}{2}$. These examples illustrate the **commutative property of addition**.

In general, we assume that for every number $a$ and every number $b$:

$$a + b = b + a$$

## Commutative Property of Multiplication

In the same way, when we multiply rational numbers, we assume that we can change the *order* of the factors without changing the product.

For example, $5 \times 4 = 4 \times 5$, and $\frac{1}{2} \times \frac{1}{4} = \frac{1}{4} \times \frac{1}{2}$. These examples illustrate the **commutative property of multiplication**.

In general, we assume that for every number $a$ and every number $b$:

$$a \times b = b \times a$$

*Subtraction* and *division* are not commutative, as shown by the following counterexample.

$$12 - 7 \neq 7 - 12 \qquad\qquad 12 \div 3 \neq 3 \div 12$$
$$5 \neq -5 \qquad\qquad 4 \neq \tfrac{3}{12}$$

## Associative Property of Addition

Addition is a binary operation; that is, we add two numbers at a time. If we wish to add three numbers, we find the sum of two and add that sum to the third. For example:

$$2 + 5 + 8 = (2 + 5) + 8 \qquad \text{or} \qquad 2 + 5 + 8 = 2 + (5 + 8)$$
$$= 7 + 8 \qquad\qquad\qquad\qquad = 2 + 13$$
$$= 15 \qquad\qquad\qquad\qquad\quad = 15$$

The way in which we *group* the numbers to be added does not change the sum. Therefore, we see that $(2 + 5) + 8 = 2 + (5 + 8)$. This example illustrates the **associative property of addition**.

In general, we assume that for every number $a$, every number $b$, and every number $c$:

$$(a + b) + c = a + (b + c)$$

## Associative Property of Multiplication

In a similar way, to find a product that involves three factors, we first multiply any two factors and then multiply this result by the third factor. We assume that we do not change the product when we change the grouping. For example:

$$5 \times 4 \times 2 = (5 \times 4) \times 2 \qquad \text{or} \qquad 5 \times 4 \times 2 = 5 \times (4 \times 2)$$
$$= 20 \times 2 \qquad\qquad\qquad\qquad = 5 \times 8$$
$$= 40 \qquad\qquad\qquad\qquad\quad = 40$$

Therefore, $(5 \times 4) \times 2 = 5 \times (4 \times 2)$. This example illustrates the **associative property of multiplication**.

In general, we assume that for every number $a$, every number $b$, and every number $c$:

$$a \times (b \times c) = (a \times b) \times c$$

*Subtraction* and *division* are not associative, as shown in the following counterexamples.

$$(15 - 4) - 3 \neq 15 - (4 - 3) \qquad\qquad (8 \div 4) \div 2 \neq 8 \div (4 \div 2)$$
$$11 - 3 \neq 15 - 1 \qquad\qquad\qquad 2 \div 2 \neq 8 \div 2$$
$$8 \neq 14 \qquad\qquad\qquad\qquad 1 \neq 4$$
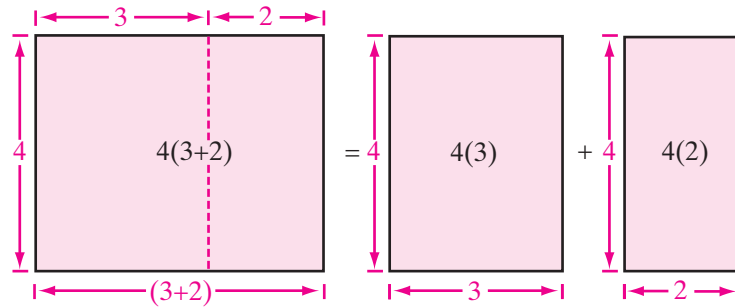
## The Distributive Property

We know $4(3 + 2) = 4(5) = 20$, and also $4(3) + 4(2) = 12 + 8 = 20$. Therefore, we see that $4(3 + 2) = 4(3) + 4(2)$.

This result can be illustrated geometrically. Recall that the area of a rectangle is equal to the product of its length and its width.

This example illustrates the **distributive property of multiplication over addition**, also called the **distributive property**. This means that the product of one number times the sum of a second and a third number equals the product of the first and second numbers plus the product of the first and third numbers.

In general, we assume that for every number *a*, every number *b*, and every number *c*:

$$a(b + c) = ab + ac \qquad \text{and} \qquad (a + b)c = ac + bc$$

The distributive property is also true for multiplication over subtraction:

$$a(b - c) = ab - ac \qquad \text{and} \qquad (a - b)c = ac - bc$$

The distributive property can be useful for mental computations. Observe how we can use the distributive property to find each of the following products as a sum:

**1.** $6 \times 23 = 6(20 + 3) = 6 \times 20 + 6 \times 3 = 120 + 18 = 138$

**2.** $9 \times 3\frac{1}{3} = 9\left(3 + \frac{1}{3}\right) = 9 \times 3 + 9 \times \frac{1}{3} = 27 + 3 = 30$

**3.** $6.5 \times 8 = (6 + 0.5)8 = 6 \times 8 + 0.5 \times 8 = 48 + 4 = 52$

Working backward, we can also use the distributive property to change the form of an expression from a sum or a difference to a product:

**1.** $5(12) + 5(8) = 5(12 + 8) = 5(20) = 100$

**2.** $7(14) - 7(4) = 7(14 - 4) = 7(10) = 70$

## Addition Property of Zero and the Additive Identity Element

The equalities $5 + 0 = 5$ and $0 + 2.8 = 2.8$ are true. They illustrate that the sum of a rational number and zero is the number itself. These examples lead us to observe that:

**1.** The **addition property of zero** states that for every number $a$:

$$a + 0 = a \quad \text{and} \quad 0 + a = a$$

**2.** The **identity element of addition**, or the **additive identity**, is 0. Thus, for any number $a$:

**If $a + x = a$, or if $x + a = a$, it follows that $x = 0$.**

## Additive Inverses (Opposites)

When we first studied integers, we learned about *opposites*. For example, the opposite of 4 is $-4$, and the opposite of $-10$ is 10.

Every rational number $a$ has an *opposite*, $-a$, such that their sum is 0, the identity element in addition. The opposite of a number is called the **additive inverse** of the number.

In general, for every rational number $a$ and its opposite $-a$:

$$a + (-a) = 0$$

On a calculator, the $\boxed{\text{(-)}}$ key, is used to enter the opposite of a number. The following example shows that the opposite of $-4.5$ is 4.5.

ENTER: $\boxed{\text{(-)}}$ $\boxed{\text{(-)}}$ 4.5 $\boxed{\textbf{ENTER}}$

DISPLAY:

```
--4.5
            4.5
```

## Multiplication Property of One and the Multiplicative Identity Element

The sentences $5 \times 1 = 5$ and $1 \times 4.6 = 4.6$ are true. They illustrate that the product of a number and one is the number itself. These examples lead us to observe that:

**1.** The **multiplication property of one** states that for every number $a$:

$$a \cdot 1 = a \quad \text{and} \quad 1 \cdot a = a$$

**2.** The **identity element of multiplication**, or the **multiplicative identity**, is **1**.

## Multiplicative Inverses (Reciprocals)

When the product of two numbers is 1 (the identity element for multiplication), then each of these numbers is called the **multiplicative inverse** or **reciprocal** of the other. Consider these examples:

$$4 \cdot \frac{1}{4} = 1$$

The reciprocal of 4 is $\frac{1}{4}$.

The reciprocal of $\frac{1}{4}$ is 4.

$$\left(2\frac{1}{2}\right)\left(\frac{2}{5}\right) = \left(\frac{5}{2}\right)\left(\frac{2}{5}\right) = 1$$

The multiplicative inverse of $2\frac{1}{2}$ or $\frac{5}{2}$ is $\frac{2}{5}$.

The multiplicative inverse of $\frac{2}{5}$ is $\frac{5}{2}$ or $2\frac{1}{2}$.

Since there is no number that, when multiplied by 0, gives 1, the number 0 has no reciprocal, or no multiplicative inverse.

In general, for every *nonzero* number $a$, there is a unique number $\frac{1}{a}$ such that:

$$a \cdot \frac{1}{a} = 1$$

On the calculator, a special key, $\boxed{x^{-1}}$ displays the reciprocal. For example, if each of the numbers shown above is entered and the reciprocal key is pressed, the reciprocal appears in decimal form.

ENTER: 4 $\boxed{x^{-1}}$ $\boxed{\text{ENTER}}$

DISPLAY:
```
4⁻¹
                    .25
```

ENTER: $\boxed{(}$ 5 $\boxed{÷}$ 2 $\boxed{)}$ $\boxed{x^{-1}}$ $\boxed{\text{ENTER}}$

DISPLAY:
```
(5/2)⁻¹
                    .4
```

**Note:** Parentheses must be used when calculating the reciprocal of a fraction.

For many other numbers, however, the decimal form of the reciprocal is not shown in its entirety in the display. For example, we know that the reciprocal of 6 is $\frac{1}{6}$, but what appears is a rational approximation of $\frac{1}{6}$.

ENTER: 6 $\boxed{x^{-1}}$ $\boxed{\text{ENTER}}$

DISPLAY:
```
6⁻¹
         .1666666667
```

The display shows the rational approximation of $\frac{1}{6}$ rounded to the last decimal place displayed by the calculator. A calculator stores more decimal places in its operating system than it has in its display. The decimal displayed times the original number will equal 1.

To display the reciprocal of 6 from the example on the previous page as a common fraction, we use 6 $\boxed{x^{-1}}$ $\boxed{\text{MATH}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$.

## Multiplication Property of Zero

The sentences $7 \times 0 = 0$ and $0 \times \frac{3}{4} = 0$ are true. They illustrate that the product of a rational number and zero is zero. This property is called the **multiplication property of zero**:

In general, for every number $a$:

$$a \cdot 0 = 0 \qquad \text{and} \qquad 0 \cdot a = 0$$

**EXAMPLE 1**

Write, in simplest form, the opposite (additive inverse) and the reciprocal (multiplicative inverse) of each of the following: **a.** 7   **b.** $-\frac{3}{8}$   **c.** $1\frac{1}{5}$   **d.** 0.2   **e.** $\pi$

**Solution**

| Number | Opposite | Reciprocal |
|---|---|---|
| **a.** 7 | $-7$ | $\frac{1}{7}$ |
| **b.** $-\frac{3}{8}$ | $\frac{3}{8}$ | $-\frac{8}{3} = -2\frac{2}{3}$ |
| **c.** $1\frac{1}{5} = \frac{6}{5}$ | $-1\frac{1}{5} = -\frac{6}{5}$ | $\frac{5}{6}$ |
| **d.** 0.2 | $-0.2$ | 5 |
| **e.** $\pi$ | $-\pi$ | $\frac{1}{\pi}$ |

**EXAMPLE 2**

Express $6t + t$ as a product and give the reason for each step.

**Solution**

| Step | Reason |
|---|---|
| (1) $6t + t = 6t + 1t$ | Multiplication property of 1. |
| (2) $\quad = (6 + 1)t$ | Distributive property. |
| (3) $\quad = 7t$ | Addition. |

*Answer*  $7t$

## EXERCISES

### Writing About Mathematics

**1.** If $x$ and $y$ represent real numbers and $xy = x$:

    **a.** What is the value of $y$ if the equation is true for all $x$? Explain your answer.

    **b.** What is the value of $x$ if the equation is true for all $y$? Explain your answer.

**2.** Cookies and brownies cost $0.75 each. In order to find the cost of 2 cookies and 3 brownies Lindsey added $2 + 3$ and multiplied the sum by $0.75. Zachary multiplied $0.75 by 2 and then $0.75 by 3 and added the products. Explain why Lindsey and Zachary both arrived at the correct cost of the cookies and brownies.

### Developing Skills

**3.** Give the value of each expression.

    **a.** $9 + 0$     **b.** $9 \times 0$     **c.** $9 \times 1$     **d.** $\frac{2}{3} \times 0$     **e.** $0 + \frac{2}{3}$     **f.** $1 \times \frac{2}{3}$

    **g.** $(\pi)\left(\frac{1}{\pi}\right)$     **h.** $1 \times 4.5$     **i.** $0 + \sqrt{7}$     **j.** $1.\overline{63} \times 0$     **k.** $1 \times \sqrt{5}$     **l.** $0 \times \sqrt{225}$

In 4–13: **a.** Replace each question mark with the number that makes the sentence true. **b.** Name the property illustrated in each sentence that is formed when the replacement is made.

    **4.** $8 + 6 = 6 + ?$               **5.** $17 \times 5 = ? \times 17$

    **6.** $(3 \times 9) \times 15 = 3 \times (9 \times ?)$     **7.** $6(5 + 8) = 6(5) + ?(8)$

    **8.** $(0.5 + 0.2) + 0.7 = 0.5 + (? + 0.7)$     **9.** $4 + 0 = ?$

    **10.** $(3 \times 7) + 5 = (? \times 3) + 5$     **11.** $(?)(8 + 2) = (8 + 2)(9)$

    **12.** $7(4 + ?) = 7(4)$            **13.** $?x = x$

In 14–25: **a.** Name the additive inverse (opposite) of each number. **b.** Name the multiplicative inverse (reciprocal) of each number.

    **14.** $17$     **15.** $1$     **16.** $-10$     **17.** $2.5$     **18.** $-1.8$     **19.** $\frac{1}{9}$

    **20.** $2\frac{1}{3}$     **21.** $-\pi$     **22.** $\frac{3}{7}$     **23.** $1.78\overline{0}$     **24.** $-\frac{1}{11}$     **25.** $3\frac{5}{71}$

In 26–31, state whether each sentence is a correct application of the distributive property. If you believe that it is not, state your reason.

    **26.** $6(5 + 8) = 6(5) + 6(8)$     **27.** $10\left(\frac{1}{2} + \frac{1}{5}\right) = 10 \times \frac{1}{2} + \frac{1}{5}$

    **28.** $5 + (8 \times 6) = (5 + 8) \times (5 + 6)$     **29.** $3(x + 5) = 3x + 3 \times 5$

    **30.** $14a - 4a = (14 - 4)a$     **31.** $18(2.5) = 18(2) + 18(0.5)$

In 32–35: **a.** Tell whether each sentence is true or false. **b.** Tell whether the commutative property holds for the given operation.

**32.** $357 + 19 = 19 + 357$

**33.** $2 \div 1 = 1 \div 2$

**34.** $25 - 7 = 7 - 25$

**35.** $18(3.6) = 3.6(18)$

In 36–39: **a.** Tell whether each sentence is true of false. **b.** Tell whether the associative property holds for the given operation

**36.** $(73 \times 68) \times 92 = 73 \times (68 \times 92)$

**37.** $(24 \div 6) \div 2 = 24 \div (6 \div 2)$

**38.** $(19 - 8) - 5 = 19 - (8 - 5)$

**39.** $9 + (0.3 + 0.7) = (9 + 0.3) + 0.7$

**40.** Insert parentheses to make each statement true.

**a.** $3 \times 2 + 1 \div 3 = 3$
**b.** $4 \times 3 \div 2 + 2 = 3$
**c.** $8 + 8 \div 8 - 8 \times 8 = 8$

**d.** $3 \div 3 + 3 \times 3 - 3 = 1$
**e.** $3 \div 3 + 3 \times 3 - 3 = 0$
**f.** $0 \times 12 \times 3 - 16 \div 8 = 0$

## Applying Skills

**41.** Steve Heinz wants to give a 15% tip to the taxi driver. The fare was $12. He knows that 10% of $12 is $1.20 and that 5% would be half of $1.20. Explain how this information can help Steve calculate the tip. What mathematical property is he using to determine the tip?

**42.** Juana rides the bus to and from work each day. Each time she rides the bus the fare is $1.75. She works five days a week. To find what she will spend on bus fare each week, Juana wants to find the product $2(1.75)(5)$. Juana rewrote the product as $2(5)(1.75)$.

**a.** What property of multiplication did Juana use when she changed $2(1.75)5$ to $2(5)(1.75)$?

**b.** What is her weekly bus fare?
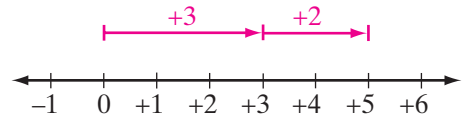
## 2-3 ADDITION OF SIGNED NUMBERS

### Adding Numbers That Have the Same Signs

The number line can be used to find the sum of two numbers. Start at 0. To add a positive number, move to the right. To add a negative number, move to the left.

**EXAMPLE 1**

Add +3 and +2.

***Solution*** Start at 0 and move 3 units to the right to +3; then move 2 more units to the right, arriving at +5.

***Calculator*** ENTER: 3 ⬚+⬚ 2 **ENTER**
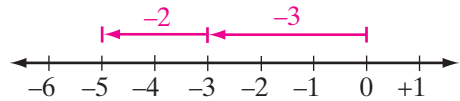***Solution***

DISPLAY:

```
3+2
                          5
```

*Answer* 5

The sum of two positive integers is the same as the sum of two whole numbers. The sum +5 is a number whose absolute value is the sum of the absolute values of +3 and +2 and whose sign is the same as the sign of +3 and +2.

**EXAMPLE 2**

Add −3 and −2.

***Solution*** Start at 0 and move 3 units to the left to −3: then move 2 more units to the left, arriving at −5.

***Calculator*** ENTER: ⬚(-)⬚ 3 ⬚+⬚ ⬚(-)⬚ 2 **ENTER**
***Solution***

DISPLAY:

```
-3+-2
                         -5
```

*Answer* −5

The sum −5 is a number whose absolute value is the sum of the absolute values of −3 and −2 and whose sign is the same as the sign of −3 and −2.

Examples 1 and 2 illustrate that the sum of two numbers with the same sign is a number whose absolute value is the sum of the absolute values of the numbers and whose sign is the sign of the numbers.

---

**Procedure**
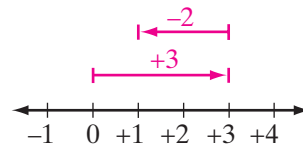
**To add two numbers that have the same sign:**

1. Find the sum of the absolute values.

2. Give the sum the common sign.

---

## Adding Numbers That Have Opposite Signs

**EXAMPLE 3**

Add: $+3$ and $-2$.

**Solution**   Start at 0 and move 3 units to the right to $+3$; then move 2 units to the left, arriving at $+1$.

**Calculator Solution**   ENTER:  3  [ + ]  [ (-) ]  2  **ENTER**

DISPLAY:

```
3 + - 2
              1
```

**Answer**   1

This sum can also be found by using properties. In the first step, substitution is used, replacing $(+3)$ with the sum $(+1) + (+2)$.
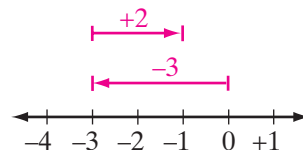
$$
\begin{aligned}
(+3) + (-2) &= [(+1) + (+2)] + (-2) && \text{Substitution} \\
&= (+1) + [(+2) + (-2)] && \text{Associative property} \\
&= +1 + 0 && \text{Addition property of opposites} \\
&= +1 && \text{Addition property of zero}
\end{aligned}
$$

The sum $+1$ is a number whose absolute value is the difference of the absolute values of $+3$ and $-2$ and whose sign is the same as the sign of $+3$, the number with the greater absolute value.

**EXAMPLE 4**

Add: $-3$ and $+2$.

**Solution**   Start at 0 and move 3 units to the left to $-3$; then move 2 units to the right, arriving at $-1$.

**Calculator Solution**   ENTER:  [ (-) ]  3  [ + ]  2  **ENTER**

DISPLAY:

```
- 3 + 2
             - 1
```

**Answer**   $-1$

This sum can also be found by using properties. In the first step, substitution is used, replacing $(-3)$ with the sum $(-1) + (-2)$.

$$
\begin{aligned}
(-3) + (+2) &= [(-1) + (-2)] + (+2) && \text{Substitution} \\
&= (-1) + [(-2) + (+2)] && \text{Associative property} \\
&= -1 + 0 && \text{Addition property of opposites} \\
&= -1 && \text{Addition property of zero}
\end{aligned}
$$

The sum $-1$ is a number whose absolute value is the difference of the absolute values of $-3$ and $+2$ and whose sign is the same as the sign of $-3$, the number with the greater absolute value.

Examples 3 and 4 illustrate that the sum of a positive number and a negative number is a number whose absolute value is the difference of the absolute values of the numbers and whose sign is the sign of the number having the larger absolute value.

---

### Procedure

**To add two numbers that have different signs:**

1. Find the difference of the absolute values of the numbers.

2. Give this difference the sign of the number that has the greater absolute value.

3. The sum is 0 if both numbers have the same absolute value.

---

**EXAMPLE 5**

Find the sum of $-3\frac{3}{4}$ and $1\frac{1}{4}$.

*Solution*          *How to Proceed*

(1) Since the numbers have different signs, find the difference of their absolute values:

$$3\frac{3}{4} - 1\frac{1}{4} = 2\frac{2}{4}$$

(2) Give the difference the sign of the number with the greater absolute value:

$$-2\frac{2}{4} = -2\frac{1}{2} \ \textit{Answer}$$

*Calculator*    The number $3\frac{3}{4}$ is the sum of the whole number 3 and the fraction $\frac{3}{4}$. The oppo-
*Solution*    site of $3\frac{3}{4}$, $-3\frac{3}{4}$, is the sum of $-3$ and $-\frac{3}{4}$.

Enclose the absolute value of the sum of 3 and $\frac{3}{4}$ in parentheses.

ENTER:   (-)   (   3   +   3   ÷   4   )   +   1   +   1   ÷   4   **ENTER**

DISPLAY:

```
-(3+3/4)+1+1/4
                -2.5
```

*Answer*    $-2.5$ or $-2\frac{1}{2}$

The method used in Example 5 to enter the negative mixed number $-3\frac{3}{4}$
illustrate the following property:

> **Property of the Opposite of a Sum**    For all real numbers $a$ and $b$:
>
> $$-(a + b) = (-a) + (-b)$$

When adding more than two signed numbers, the commutative and associa-
tive properties allow us to arrange the numbers in any order and to group them
in any way. It may be helpful to add positive numbers first, add negative num-
bers next, and then add the two results.

## EXERCISES

### Writing About Mathematics

**1.** The sum of two numbers is positive. One of the numbers is a positive number that is larger
than the sum. Is the other number positive or negative? Explain your answer.

**2.** The sum of two numbers is positive. One of the numbers is negative. Is the other number
positive or negative? Explain your answer.

**3.** The sum of two numbers is negative. One of the numbers is a negative number that is
smaller than the sum. Is the other number positive or negative? Explain your answer.

**4.** The sum of two numbers is negative. One of the numbers is positive. Is the other number
positive or negative? Explain your answer.

**5.** Is it possible for the sum of two numbers to be smaller than either of the numbers? If so,
give an example.

### Developing Skills

In 6–10, find each sum or difference.

**6.** $|-6| + |4|$        **7.** $|-10| - |-5|$        **8.** $|4.5| - |-4.5|$        **9.** $|+6| + |-4|$        **10.** $|+6 - 4|$

In 11–27, add the numbers. Use a calculator to check your answer.

**11.** $-17 + (-28)$   **12.** $+23 + (-35)$   **13.** $-87 + (+87)$   **14.** $-2.06 + 1.37$

**15.** $-33\frac{1}{3} + 19\frac{2}{3}$   **16.** $-5\frac{3}{4} + 8\frac{1}{2}$   **17.** $+7 + \left(-6\frac{2}{7}\right)$   **18.** $(-3.72) + (-5.28)$

**19.** $(-47) + (-35) + (+47)$   **20.** $34\frac{3}{8} + (-73)$   **21.** $-73\frac{1}{2} + 86$

**22.** $-14\frac{3}{4} + \left(-17\frac{3}{4}\right)$   **23.** $\left|-13\right| + (-13)$   **24.** $\left|-42\right| + (-\left|43\right|)$

**25.** $-6.25 + (-0.75)$   **26.** $(-12.4) + 13.0$   **27.** $\left|-12.4\right| + 13.0$

## Applying Skills

**28.** In 1 hour, the temperature rose 4° Celsius and in the next hour it dropped 6° Celsius. What was the net change in temperature during the two-hour period?

**29.** An elevator started on the first floor and rose 30 floors. Then it came down 12 floors. At which floor was it at that time?

**30.** A football team gained 7 yards on the first play, lost 2 yards on the second, and lost 8 yards on the third. What was the net result of the three plays?

**31.** Fay has $250 in a bank. During the month, she made a deposit of $60 and a withdrawal of $80. How much money did Fay have in the bank at the end of the month?

**32.** During a four-day period, the dollar value of a share of stock rose $1.50 on the first day, dropped $0.85 on the second day, rose $0.12 on the third day, and dropped $1.75 on the fourth day. What was the net change in the stock during this period?

## 2-4 SUBTRACTION OF SIGNED NUMBERS

In arithmetic, to subtract 3 from 7, we find the number that, when added to 3, gives 7. We know that $7 - 3 = 4$ because $3 + 4 = 7$. Subtraction is the inverse operation of addition.

**DEFINITION**

In general, for every number $c$ and every number $b$, the expression $c - b$ is the number $a$ such that $b + a = c$.

We use this definition in order to subtract signed numbers. To subtract $-2$ from $+3$, written as $(+3) - (-2)$, we must find a number that, when added to $-2$, will give $+3$. We write:

$$(-2) + (?) = +3$$

We can use a number line to find the answer to this open sentence. From a point 2 units to the left of 0, move to the point that represents +3, that is, 3 units to the right of 0. We move 5 units to the right, a motion that represents +5.
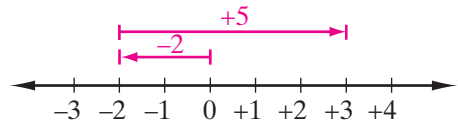


Therefore, $(+3) - (-2) = +5$ because $(-2) + (+5) = +3$.

Notice that $(+3) - (-2)$ can also be represented as the directed distance from −2 to +3 on the number line.

Subtraction can be written vertically as follows:

$$
\begin{array}{lll}
(+3) & \quad or \quad Subtract: (+3) & \text{minuend} \\
\underline{(-2)} & \quad\quad\quad\quad\quad \underline{(-2)} & \text{subtrahend} \\
+5 & \quad\quad\quad\quad\quad +5 & \text{difference}
\end{array}
$$

When you first learned to subtract numbers, you learned to check your answer. The answer (the difference) plus the number being subtracted (the subtrahend) must be equal to the number from which you are subtracting (the minuend).

Check each of the following examples using:

subtrahend + difference = minuend:

$$
\begin{array}{lcccc}
Subtract: & +9 & -7 & +5 & -3 \\
& \underline{+6} & \underline{-2} & \underline{-2} & \underline{+1} \\
& +3 & -5 & +7 & -4
\end{array}
$$

Now, consider another way in which addition and subtraction are related. In each of the following examples, compare the result obtained when a signed number is subtracted with the result obtained when the opposite of that signed number is added.

| Subtract | Add | Subtract | Add | Subtract | Add | Subtract | Add |
|----------|-----|----------|-----|----------|-----|----------|-----|
| +9 | +9 | −7 | −7 | +5 | +5 | −3 | −3 |
| +6 | −6 | −2 | +2 | −2 | +2 | +1 | −1 |
| +3 | +3 | −5 | −5 | +7 | +7 | −4 | −4 |

Observe that, in each example, adding the opposite (the additive inverse) of a signed number gives the same result as subtracting that signed number. It therefore seems reasonable to define subtraction as follows:

**DEFINITION**

If $a$ is any signed number and $b$ is any signed number, then:

$$a - b = a + (-b)$$

### Procedure

**To subtract one signed number from another, add the opposite (additive inverse) of the subtrahend to the minuend.**

## **Uses of the Symbol −**

We have used the symbol − in three ways:

**1.** To indicate that a number is negative:   $-2$      Negative 2

**2.** To indicate the opposite of a number:   $-(-4)$    Opposite of negative 4

                                     $-a$        Opposite of $a$

**3.** To indicate subtraction:               $4-(-3)$  Difference between 4 and $-3$

Note that an arithmetic expression such as $+3 - 7$ can mean either the difference between $+3$ and $+7$ or the sum of $+3$ and $-7$.

$$+3 - 7 = +3 - (+7) = +3 + (-7)$$

When writing an arithmetic expression, we use the same sign for both a negative number and subtraction. On a calculator, the key for subtraction, $\boxed{-}$, is not the same key as the key for a negative number, $\boxed{(-)}$. Using the wrong key will result in an error message.

**EXAMPLE I**

Perform the indicated subtractions.

**a.** $(+30) - (+12)$    **b.** $(-19) - (-7)$    **c.** $(-4) - (0)$    **d.** $0 - 8$

*Answers*  **a.** $+18$              **b.** $-12$            **c.** $-4$         **d.** $-8$

**EXAMPLE 2**

From the sum of $-2$ and $+8$, subtract $-5$.

*Solution*  $(-2 + 8) - (-5) = +6 + (+5) = +11$

*Calculator Solution*  ENTER: $\boxed{(-)}$ 2 $\boxed{+}$ 8 $\boxed{-}$ $\boxed{(-)}$ 5 $\boxed{\text{ENTER}}$

DISPLAY:
```
-2+8--5
              11
```

*Answer*  11

**EXAMPLE 3**

Subtract the sum of $-7$ and $+2$ from $-4$.

**Solution**   $-4 - (-7 + 2) = -4 - (-5) = -4 + (+5) = +1$

**Calculator**   ENTER:   (-)  4   −   (   (-)  7   +   2   )   **ENTER**
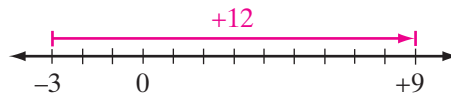**Solution**
DISPLAY:
```
-4-(-7+2)
                    1
```

**Answer**   1

**EXAMPLE 4**

How much greater than $-3$ is 9?

**Solution**   $9 - (-3) = 9 + 3 = 12$

**Answer**   9 is 12 greater than $-3$.

Note that parentheses need not be entered in the calculator in Example 2 since the additions and subtractions are to be done in the order in which they occur in the expression. Parentheses are needed, however, in Example 3 since the sum of $-7$ and $+2$ is to be found first, before subtracting this sum from $-4$.

## EXERCISES

**Writing About Mathematics**

**1.** Does $+8 - 12$ mean the difference between $+8$ and 12 or the sum of $+8$ and $-12$? Explain your answer.

**2.** How is addition used to check subtraction?

**Developing Skills**

In 3–10, perform each indicated subtraction. Check your answers using a calculator.

**3.** $-23 - (-35)$      **4.** $-87 - (+87)$      **5.** $+5.4 - (-8.6)$      **6.** $-8.8 - (-3.7)$

**7.** $-2.06 - (+1.37)$      **8.** $-33\frac{1}{3} - \left(+19\frac{2}{3}\right)$      **9.** $-5\frac{3}{4} - \left(-8\frac{1}{2}\right)$      **10.** $+7 - \left(6\frac{2}{7}\right)$

In 11–18, Find the value of each given expression.

**11.** $(+18) - (-14)$        **12.** $(-3.72) - (-5.28)$        **13.** $(-12) + (-57 - 12)$

**14.** $(-47) - (-35 + 47)$      **15.** $-73\frac{1}{2} - 86$        **16.** $-14\frac{3}{4} - \left(-17\frac{3}{4}\right)$

**17.** $-34\frac{3}{8} - \left(-73 - 72\frac{3}{8}\right)$      **18.** $|-32| - (-32)$

**19.** How much is 18 decreased by $-7$?

**20.** How much greater than $-15$ is 12?

**21.** How much greater than $-4$ is $-1$?

**22.** What number is 6 less than $-6$?

**23.** Subtract 8 from the sum of $-6$ and $-12$.

**24.** Subtract $-7$ from the sum of $+18$ and $-10$.

**25.** State whether each of the following sentences is true or false:

    **a.** $(+5) - (-3) = (-3) - (+5)$        **b.** $(-7) - (-4) = (-4) - (-7)$

**26.** If $x$ and $y$ represent real numbers:

    **a.** Does $x - y = y - x$ for all replacements of $x$ and $y$? Justify your answer.

    **b.** Does $x - y = y - x$ for any replacements of $x$ and $y$? For which values of $x$ and $y$?

    **c.** What is the relation between $x - y$ and $y - x$ for all replacements of $x$ and $y$?

    **d.** Is the operation of subtraction commutative? In other words, for all signed numbers $x$ and $y$, does $x - y = y - x$?

**27.** State whether each of the following sentences is true or false:

    **a.** $(15 - 9) - 6 = 15 - (9 - 6)$        **b.** $[(-10) - (+4)] - (+8) = (-10) - [(+4) - (+8)]$

**28.** Is the operation of subtraction associative? In other words, for all signed numbers $x$, $y$, and $z$, does $(x - y) - z = x - (y - z)$? Justify your answer.

## Applying Skills

**29.** Express as a signed number the increase or decrease when the Celsius temperature changes from:

    **a.** $+5°$ to $+8°$        **b.** $-10°$ to $+18°$        **c.** $-6°$ to $-18°$        **d.** $+12°$ to $-4°$

**30.** Find the change in altitude when you go from a place that is 15 meters below sea level to a place that is 95 meters above sea level.

**31.** In a game, Sid was 35 points "in the hole." How many points must he make in order to have a score of 150 points?

**32.** The record high Fahrenheit temperature in New City is $105°$; the record low is $-9°$. Find the difference between these temperatures.

**33.** At one point, the Pacific Ocean is 0.50 kilometers in depth; at another point it is 0.25 kilometers in depth. Find the difference between these depths.

## 2-5 MULTIPLICATION OF SIGNED NUMBERS

### Four Possible Cases in the Multiplication of Signed Numbers

We will use a common experience to illustrate the various cases that can arise in the multiplication of signed numbers.

**1.** Represent a gain in weight by a positive number and a loss of weight by a negative number.

**2.** Represent a number of months in the future by a positive number and a number of months in the past by a negative number.

**CASE 1**  *Multiplying a Positive Number by a Positive Number*

If a boy gains 2 pounds each month, 4 months from now he will be 8 pounds heavier than he is now. Using signed numbers, we may write:

$$(+2)(+4) = +8$$

The product of the two positive numbers is a positive number.

**CASE 2**  *Multiplying a Negative Number by a Positive Number*

If a boy loses 2 pounds each month, 4 months from now he will be 8 pounds lighter than he is now. Using signed numbers, we may write:

$$(-2)(+4) = -8$$

The product of the negative number and the positive number is a negative number.

**CASE 3**  *Multiplying a Positive Number by a Negative Number*

If a girl gained 2 pounds each month, 4 months ago she was 8 pounds lighter than she is now. Using signed numbers, we may write:

$$(+2)(-4) = -8$$

The product of the positive number and the negative number is a negative number.

**CASE 4**  *Multiplying a Negative Number by a Negative Number*

If a girl lost 2 pounds each month, 4 months ago she was 8 pounds heavier than she is now. Using signed numbers, we may write:

$$(-2)(-4) = +8$$

The product of the two negative numbers is a positive number.

In all four cases, the absolute value of the product, 8, is equal to the product of the absolute values of the factors, 4 and 2.

## Using the Properties of Real Numbers to Multiply Signed Numbers

You know that the sum of any real number and its inverse is 0, $a + (-a) = 0$, and that for all real numbers $a$, $b$, and $c$, $a(b + c) = ab + ac$.

These two facts will enable us to demonstrate the rules for multiplying signed numbers.

$$4[7 + (-7)] = 4(7) + 4(-7)$$
$$4(0) = 28 + 4(-7)$$
$$0 = 28 + 4(-7)$$

In order for this to be a true statement, $4(-7)$ must equal $-28$, the opposite of 28.

Since addition is a commutative operation, $4(-7) = -7(4)$. Then

$$-7[4 + (-4)] = -7(4) + (-7)(-4)$$
$$-7(0) = -28 + (-7)(-4)$$
$$0 = -28 + (-7)(-4)$$

In order for this to be a true statement, $-7(-4)$ must be 28, the additive inverse of $-28$.

### Rules for Multiplying Signed Numbers

**RULE 1** The product of two positive numbers or of two negative numbers is a positive number whose absolute value is the product of the absolute values of the numbers.

**RULE 2** The product of a positive number and a negative number is a negative number whose absolute value is the product of the absolute values of the numbers.

In general, if $a$ and $b$ are both positive or are both negative, then:

$$ab = |a| \cdot |b|$$

If one of the numbers, $a$ or $b$, is positive and the other is negative, then:

$$ab = -(|a| \cdot |b|)$$

**Procedure**

**To multiply two signed numbers:**

1. Find the product of the absolute values.

2. Write a plus sign before this product when the two numbers have the same sign.

3. Write a minus sign before this product when the two numbers have different signs.

**EXAMPLE 1**

Find the product of each of the given pairs of numbers.

|  | *Answers* |  | *Answers* |
|---|---|---|---|
| **a.** $(+12)(+4)$ | $= +48$ | **b.** $(-13)(-5)$ | $= +65$ |
| **c.** $(+18)(-3)$ | $= -54$ | **d.** $(-15)(+6)$ | $= -90$ |
| **e.** $(+3.4)(-3)$ | $= -10.2$ | **f.** $\left(-7\frac{1}{8}\right)(-3)$ | $= +21\frac{3}{8}$ |

**EXAMPLE 2**

Use the distributive property of multiplication over addition to find the product $8(-72)$.

*Solution*
$$8(-72) = 8[(-70) + (-2)]$$
$$= 8(-70) + 8(-2)$$
$$= (-560) + (-16)$$
$$= -576 \ \textit{Answer}$$

**EXAMPLE 3**

Find the value of $(-2)^3$.

*Solution*
$$(-2)^3 = (-2)(-2)(-2) = +4(-2) = -8$$

*Answer*  $-8$

**Note:** The product of an odd number (3) of negative factors is negative.

**EXAMPLE 4**

Find the value of $(-3)^4$.

*Solution*   $(-3)^4 = (-3)(-3)(-3)(-3) = [(-3)(-3)][(-3)(-3)] = (+9)(+9) = +81$

*Answer*  $+81$

**Note**: The product of an even number (4) of negative factors is positive.

In this example, the value of $(-3)^4$ was found to be $+81$. This is not equal to $-3^4$, which is the opposite of $3^4$ or $-1(3^4)$. To find the value of $-3^4$, first find the value of $3^4$, which is 81, and then write the opposite of this power, $-81$. Thus, $(-3)^4 = 81$ and $-3^4 = -81$.

On a calculator, evaluate $(-3)^4$.

ENTER: ( ) (-) 3 ) ^ 4 **ENTER**

DISPLAY:
```
(-3)^4
              81
```

On a calculator, evaluate $-3^4$.

ENTER: (-) 3 ^ 4 **ENTER**

DISPLAY:
```
-3^4
             -81
```

## EXERCISES

### Writing About Mathematics

**1.** Javier said that $(-5)(-4)(-2) = +40$ because the product of numbers with the same sign is positive. Explain to Javier why he is wrong.

**2.** If $-a(b)$ means the opposite of $ab$, explain how knowing that $3(-4) = -12$ can be used to show that $-3(-4) = +12$.

### Developing Skills

In 3–12, find the product of each pair of numbers. Check your answers using a calculator.

**3.** $-17(-6)$     **4.** $+27(-6)$     **5.** $-9(+27)$     **6.** $-23(-15)$     **7.** $+4(-4)$

**8.** $+5.4(-0.6)$     **9.** $-2.6(+0.05)$     **10.** $-3\frac{1}{3}\left(+1\frac{2}{3}\right)$     **11.** $-5\frac{3}{4}\left(-\frac{1}{2}\right)$     **12.** $-6\frac{2}{3}(7)$

In 13–18, find the value of each expression.

**13.** $(+18)(-4)(-5)$     **14.** $(-3.72)(-0.5)(+0.2)$     **15.** $(-4)(-35 + 7)$

**16.** $(-12)(-7 - 3)$     **17.** $-8\left(-73 + 72\frac{3}{8}\right)$     **18.** $-4\left|-\frac{3}{4} + \frac{1}{4}\right|$

In 19–28, find the value of each power.

**19.** $(-3)^2$     **20.** $-3^2$     **21.** $(-5)^3$     **22.** $(+2)^4$     **23.** $(-2)^4$

**24.** $-2^4$     **25.** $(0.5)^2$     **26.** $(-0.5)^2$     **27.** $-0.5^2$     **28.** $-(-3)^3$

In 29–34, name the property that is illustrated in each statement.

**29.** $2(-1 + 3) = 2(-1) + 2(3)$     **30.** $(-2) + (-1) = (-1) + (-2)$

**31.** $(-1 + 2) + 3 = -1 + (2 + 3)$     **32.** $-2 + 0 = -2$

**33.** $(-5)(7) = (7)(-5)$     **34.** $3 \times (2 \times -3) = (3 \times 2) \times (-3)$

## 2-6 DIVISION OF SIGNED NUMBERS

### Using the Inverse Operation in Dividing Signed Numbers

Division may be defined as the inverse operation of multiplication, just as subtraction is defined as the inverse operation of addition. To divide 6 by 2 means to find a number that, when multiplied by 2, gives 6. That number is 3 because $3(2) = 6$. Thus, $\frac{6}{2} = 3$, or $6 \div 2 = 3$. The number 6 is the dividend, 2 is the divisor, and 3 is the quotient.

It is impossible to divide a signed number by 0; that is, division by 0 is undefined. For example, to solve $(-9) \div 0$, we would have to find a number that, when multiplied by 0, would give $-9$. There is no such number since the product of any signed number and 0 is 0.

▶ **In general, for all signed numbers $a$ and $b$ ($b \neq 0$), $a \div b$ or $\frac{a}{b}$ means to find the unique number $c$ such that $cb = a$**

In dividing nonzero signed numbers, there are four possible cases. Consider the following examples:

**CASE 1** $\frac{+6}{+3} = ?$ implies $(?)(+3) = +6$. Since $(+2)(+3) = +6$, $\frac{+6}{+3} = +2$

**CASE 2** $\frac{-6}{-3} = ?$ implies $(?)(-3) = -6$. Since $(+2)(-3) = -6$, $\frac{-6}{-3} = +2$

**CASE 3** $\frac{-6}{+3} = ?$ implies $(?)(+3) = -6$. Since $(-2)(+3) = -6$, $\frac{-6}{+3} = -2$

**CASE 4** $\frac{+6}{-3} = ?$ implies $(?)(-3) = +6$. Since $(-2)(-3) = +6$, $\frac{+6}{-3} = -2$

In the preceding examples, observe that:

1. When the dividend and divisor are both positive or both negative, the quotient is positive.

2. When the dividend and divisor have opposite signs, the quotient is negative.

3. In all cases, the absolute value of the quotient is the absolute value of the dividend divided by the absolute value of the divisor.

### Rules for Dividing Signed Numbers

**RULE 1** The quotient of two positive numbers or of two negative numbers is a positive number whose absolute value is the absolute value of the dividend divided by the absolute value of the divisor.

**RULE 2** The quotient of a positive number and a negative number is a nega-
tive number whose absolute value is the absolute value of the dividend
divided by the absolute value of the divisor.

In general, if $a$ and $b$ are both positive or are both negative, then:

$$a \div b = |a| \div |b| \quad \text{or} \quad \frac{a}{b} = \frac{|a|}{|b|}$$

If one of the numbers, $a$ or $b$, is positive and the other is negative, then:

$$a \div b = -(|a| \div |b|) \quad \text{or} \quad \frac{a}{b} = -\left(\frac{|a|}{|b|}\right)$$

### Procedure

**To divide two signed numbers:**

1. Find the quotient of the absolute values.

2. Write a plus sign before this quotient when the two numbers have the same sign.

3. Write a minus sign before this quotient when the two numbers have different signs.

### Rule for Dividing Zero by a Nonzero Number

If the expression $0 \div (-5)$ or $\frac{0}{-5} = ?$, then $(?)(-5) = 0$. Since 0 is the only num-
ber that can replace ? and result in a true statement, $0 \div (-5)$ or $\frac{0}{-5} = 0$. This
illustrates that 0 divided by any nonzero number is 0.

In general, if $a$ is a nonzero number ($a \neq 0$), then

$$0 \div a = \frac{0}{a} = 0.$$

**EXAMPLE 1**

Perform each indicated division, if possible.

**a.** $\frac{+60}{+15}$   **b.** $\frac{+10}{-90}$   **c.** $\frac{-27}{-3}$   **d.** $(-45) \div 9$   **e.** $0 \div (-9)$   **f.** $-3 \div 0$

*Answers*   **a.** $+4$   **b.** $-\frac{1}{9}$   **c.** $+9$   **d.** $-5$   **e.** $0$   **f.** Undefined

## Using the Reciprocal in Dividing Signed Numbers

In Section 2-2, we learned that for every nonzero number $a$, there is a unique
number $\frac{1}{a}$, called the *reciprocal* or *multiplicative inverse*, such that $a \cdot \frac{1}{a} = 1$.

Using the reciprocal of a number, we can define division in terms of multiplication as follows:

For all numbers $a$ and $b$ $(b \neq 0)$:

$$a \div b = \tfrac{a}{b} = a \cdot \tfrac{1}{b} \ (b \neq 0)$$

### Procedure

**To divide a signed number by a nonzero signed number, multiply the dividend by the reciprocal of the divisor.**

Notice that we exclude division by 0. The set of nonzero real numbers is closed with respect to division because every nonzero real number has a unique reciprocal, and multiplication by this reciprocal is always possible.

**EXAMPLE 2**

Perform each indicated division by using the reciprocal of the divisor.

*Answers*

**a.** $\dfrac{+30}{+2}$ $\qquad = (+30)\left(+\tfrac{1}{2}\right) = +15$

**b.** $\dfrac{-30}{+90}$ $\qquad = (-30)\left(+\tfrac{1}{90}\right) = -\tfrac{1}{3}$

**c.** $(-54) \div 6$ $\qquad = (-54)\left(\tfrac{1}{6}\right) = -9$

**d.** $-27 \div \left(-\tfrac{1}{3}\right)$ $\qquad = -27 \times \left(-\tfrac{3}{1}\right) = +81$

**e.** $(+3) \div \left(-\tfrac{3}{5}\right)$ $\qquad = +3\left(-\tfrac{5}{3}\right) = -5$

**f.** $0 \div (-9)$ $\qquad = 0\left(-\tfrac{1}{9}\right) = 0$

## EXERCISES

### Writing About Mathematics

**1.** If $x$ and $y$ represent nonzero numbers, what is the relationship between $x \div y$ and $y \div x$?

**2.** If $x \neq y$, are there any values of $x$ and $y$ for which $x \div y = y \div x$?

### Developing Skills

In 3–10, name the reciprocal (the multiplicative inverse) of each given number.

**3.** $+6$ $\qquad$ **4.** $-5$ $\qquad$ **5.** $1$ $\qquad\qquad$ **6.** $-1$

**7.** $\tfrac{1}{2}$ $\qquad$ **8.** $-\tfrac{1}{10}$ $\qquad$ **9.** $-\tfrac{3}{4}$ $\qquad$ **10.** $x$ if $x \neq 0$

In 11–26, find the indicated quotients or write "undefined" if no quotient exists.

**11.** $\frac{-63}{-9}$       **12.** $\frac{-48}{+16}$       **13.** $\frac{-10}{+10}$       **14.** $\frac{0}{-13}$

**15.** $\frac{-15}{-45}$       **16.** $\frac{+3.6}{-0.12}$       **17.** $\frac{-0.25}{-2.5}$       **18.** $\frac{+0.01}{-0.001}$

**19.** $(+100) \div (-2.5)$       **20.** $(-75) \div (0)$

**21.** $(+0.5) \div (-0.25)$       **22.** $(-1.5) \div (-0.03)$

**23.** $(+12) \div \left(-\frac{1}{3}\right)$       **24.** $\left(-\frac{3}{4}\right) \div (+6)$

**25.** $\left(+\frac{7}{8}\right) \div \left(-\frac{21}{32}\right)$       **26.** $\left(-1\frac{1}{4}\right) \div \left(-2\frac{1}{2}\right)$

In 27–28, state whether each sentence is true or false:

**27. a.** $[(+16) \div (+4)] \div (+2) = (+16) \div [(+4) \div (+2)]$

    **b.** $[(-36) \div (+6)] \div (-2) = (-36) \div [(+6) \div (-2)]$

    **c.** Division is associative.

**28. a.** $(12 + 6) \div 2 = 12 \div 2 + 6 \div 2$

    **b.** $[(+25) + (-10)] \div (-5) = (+25) \div (-5) + (-10) \div (-5)$

    **c.** $2 \div (3 + 5) = 2 \div 3 + 2 \div 5$

    **d.** Division is distributive over addition and subtraction.

## 2-7 OPERATIONS WITH SETS

Recall that a set is simply a collection of distinct objects or elements, such as a set of numbers in arithmetic or a set of points in geometry. And, just as there are operations in arithmetic and in geometry, there are operations with sets. Before we look at these operations, we need to understand one more type of set.

The **universal set**, or the **universe**, is the set of all elements under consideration in a given situation, usually denoted by the letter $U$. For example:

**1.** Some universal sets, such as all the numbers we have studied, are infinite. Here, $U = \{$real numbers$\}$.

**2.** In other situations, such as the scores on a classroom test, the universal set can be finite. Here, using whole-number grades, $U = \{0, 1, 2, 3, \ldots, 100\}$.

Three operations with sets are called *intersection*, *union*, and *complement*.

### Intersection of Sets

The **intersection of two sets**, $A$ and $B$, denoted by $A \cap B$, is the set of all elements that belong to both sets, $A$ and $B$. For example:

1. When $A = \{1, 2, 3, 4, 5\}$, and $B = \{2, 4, 6, 8, 10\}$, then $A \cap B$ is $\{2, 4\}$.

2. In the diagram, two lines called $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect. Each line is an infinite set of points although only three points are marked on each line. The intersection is a set that has one element, point $E$, the point that is on line $AB$ ($\overleftrightarrow{AB}$) and on line $CD$ ($\overleftrightarrow{CD}$). We write the intersection of the lines in the example shown as $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = E$.
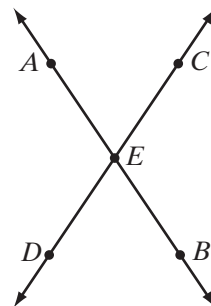
3. Intersection is a binary operation, $F \cap G = H$, where sets $F$, $G$, and $H$ are subsets of the universal set and $\cap$ is the operation symbol. For example:

$$U = \text{set of natural numbers} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, \ldots\}$$
$$F = \text{multiples of } 2 = \{2, 4, 6, 8, 10, 12, \ldots\}$$
$$G = \text{multiples of } 3 = \{3, 6, 9, 12, 15, 18, \ldots\}$$
$$F \cap G = \text{multiples of } 6 = \{6, 12, 18, 24, 30, \ldots\}$$

4. Two sets are **disjoint sets** if their intersection is the **empty set** ($\varnothing$ or $\{\}$); that is, if they do not have a common element. For example, when $K = \{1, 3, 5, 7, 9, 11, 13\}$ and $L = \{2, 4, 6, 8\}$, $K \cap L = \varnothing$. Therefore, $K$ and $L$ are disjoint sets.

## Union of Sets

The **union of two sets**, $A$ and $B$, denoted by $A \cup B$, is the set of all elements that belong to set $A$ or to set $B$, or to both set $A$ and set $B$. For example:

1. If $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6\}$, then $A \cup B = \{1, 2, 3, 4, 6\}$. Note that an element is not repeated in the union of two sets even if it is an element of each set.

2. In the diagram, both region $R$ (gray shading) and region S (light color shading) represent sets of points. The shaded parts of both regions represents $R \cup S$, and the dark color shading where the regions overlap represents $R \cap S$.
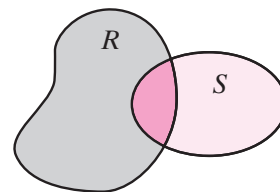
3. If $A = \{1, 2\}$ and $B = \{1, 2, 3, 4, 5\}$, then the union of $A$ and $B$ is $\{1, 2, 3, 4, 5\}$. We can write $A \cup B = \{1, 2, 3, 4, 5\}$, or $A \cup B = B$. Once again we have an example of a binary operation, where the elements are taken from a universal set and where the operation here is union.

**4.** The union of the set of all rational numbers and the set of all irrational numbers is the set of real numbers.

**{real numbers} = {rational numbers} ∪ {irrational numbers}**

## Complement of a Set

The **complement of a set** $A$, denoted by $\overline{A}$, is the set of all elements that belong to the universe $U$ but do not belong to set $A$. Therefore, before we can determine the complement of $A$, we must know $U$. For example:

**1.** If $A = \{3, 4, 5\}$ and $U = \{1, 2, 3, 4, 5\}$, then $\overline{A} = \{1, 2\}$ because 1 and 2 belong to the universal set $U$ but do not belong to set $A$.

**2.** If the universe is {whole numbers} and $A$ = {even whole numbers} then $\overline{A}$ = {odd whole numbers} because the odd whole numbers belong to the universal set but do not belong to set $A$.

Although it seems at first that only one set is being considered in writing the complement of $A$ as $\overline{A}$, actually there are two sets. This fact suggests a binary operation, in which the universe $U$ and the set $A$ are the pair of elements, *complement* is the operation, and the unique result is $\overline{A}$. The complement of any universe is the empty set. Note that the complement can also be written as $U \setminus A$ to emphasize that it is a binary operation.

**EXAMPLE I**

If $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{6, 7\}$, and $B = \{3, 5, 7\}$, determine $\overline{A} \cap \overline{B}$.

*Solution*
$$U = \{1, 2, 3, 4, 5, 6, 7\}$$

Since $A = \{6, 7\}$, then $\overline{A} = \{1, 2, 3, 4, 5\}$.

Since $B = \{3, 5, 7\}$, then $\overline{B} = \{1, 2, 4, 6\}$.

Since 1, 2, and 4 are elements in both $\overline{A}$ and $\overline{B}$, we can write:
$$\overline{A} \cap \overline{B} = \{1, 2, 4\} \; \textit{Answer}$$

**EXAMPLE 2**

Using sets $U$, $A$, and $B$ given for Example 1, find the complement of the set $A \cup B$, that is, determine $\overline{A \cup B}$.

*Solution*  Since $A \cup B$ contains all of the elements that are common to $A$ and $B$, $A \cup B = \{3, 5, 6, 7\}$. Therefore,
$$\overline{A \cup B} = \{1, 2, 4\} \; \textit{Answer}$$

## EXERCISES

### Writing About Mathematics

**1.** A line is a set of points. Can the intersection of two lines be the empty set? Explain.

**2.** Is the union of the set of prime numbers and the set of composite numbers equal to the set of counting numbers? Explain.

### Developing Skills

In 3–10, $A = \{1, 2, 3\}$, $B = \{3, 4, 5, 6\}$, and $C = \{1, 3, 4, 6\}$. In each case, perform the given operation and list the element(s) of the resulting set.

**3.** $A \cap B$      **4.** $A \cup B$      **5.** $A \cap C$      **6.** $A \cup C$

**7.** $B \cap C$      **8.** $B \cup C$      **9.** $B \cup \varnothing$      **10.** $B \cap \varnothing$

**11.** Using the sets $A$, $B$, and $C$ given for Exercises 3–10, list the element(s) of the smallest possible universal set of which $A$, $B$, and $C$ are all subsets.

In 12–19, the universe $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 5\}$, $B = \{2, 5\}$, and $C = \{2\}$. In each case, perform the given operation and list the element(s) of the resulting set.

**12.** $\overline{A}$      **13.** $\overline{B}$      **14.** $\overline{C}$      **15.** $A \cup B$

**16.** $A \cap B$      **17.** $A \cup \overline{B}$      **18.** $\overline{A} \cap \overline{B}$      **19.** $\overline{A} \cup \overline{B}$

**20.** If $U = \{2, 4, 6, 8\}$ and $\overline{A} = \{6\}$, what are the elements of $A$?

**21.** If $U = \{2, 4, 6, 8\}$, $A = \{2\}$, and $\overline{B} = \{2, 4\}$, what are the elements of $A \cup B$?

**22.** If $U = \{2, 4, 6, 8\}$, $A = \{2\}$, and $\overline{B} = \{2, 4\}$, what is the set $A \cap B$?

**23.** Suppose that the set $A$ has two elements and the set $B$ has three elements.

     **a.** What is the greatest number of elements that $A \cup B$ can have?

     **b.** What is the least number of elements that $A \cup B$ can have?

     **c.** What is the greatest number of elements that $A \cap B$ can have?

     **d.** What is the least number of elements that $A \cap B$ can have?

**24.** Let the universe $U = \{2, 4, 6, 8, 10, 12\}$, $A = \{2, 8, 12\}$ and $B = \{4, 10\}$.

     **a.** $\overline{A}$      **b.** $\overline{\overline{A}}$ (the complement of $\overline{A}$)      **c.** $\overline{B}$      **d.** $\overline{\overline{B}}$ (the complement of $\overline{B}$)

**25.** Let the universe $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

     **a.** Find the elements of $A \cap B$, $\overline{A} \cap \overline{B}$, and $\overline{A} \cup \overline{B}$ when $A$ and $B$ are equal to:

         (1) $A = \{1, 2, 3, 4\}$; $B = \{5, 6, 7, 8\}$          (2) $A = \{2, 4\}$; $B = \{6, 8\}$

         (3) $A = \{1, 3, 5, 7\}$; $B = \{2, 4, 6, 8\}$          (4) $A = \{2\}$; $B = \{4\}$

     **b.** When $A$ and $B$ are disjoint sets, describe, in words, the set $\overline{A} \cap \overline{B}$.

     **c.** If $A$ and $B$ are disjoint sets, what is the set $\overline{A} \cup \overline{B}$? Explain.

## 2-8 GRAPHING NUMBER PAIRS

Even though we know that the surface of the earth is approximately the surface of a sphere, we often model the earth by using maps that are plane surfaces. To locate a place on a map, we choose two reference lines, the equator and the prime meridian. The location of a city is given in term of east or west *longitude* (distance from the prime meridian) and north or south *latitude* (distance from the equator). For example, the city of Lagos in Nigeria is located at 3° east longitude and 6° north latitude, and the city of Dakar in Senegal is located 17° west longitude and 15° north latitude.
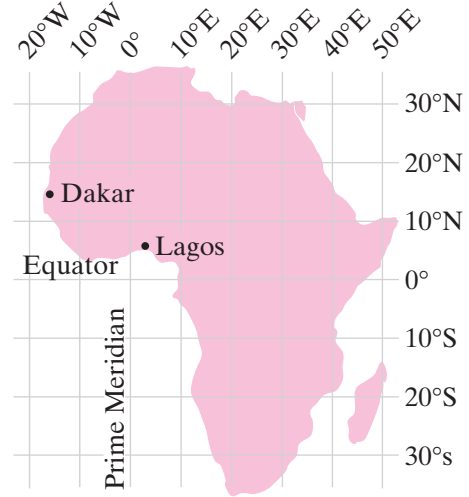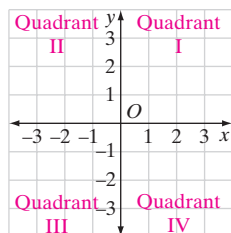
### Points on a Plane

The method used to locate cities on a map can be used to locate any point on a plane. The reference lines are a horizontal number line called the **x-axis** and a vertical number line called the **y-axis**. These two number lines, which have the same scale and are drawn perpendicular to each other, are called the **coordinate axes**. The plane determined by the axes is called the **coordinate plane**.

In a coordinate plane, the intersection of the two axes is called the **origin** and is indicated as point $O$. This point of intersection is assigned the value 0 on both the $x$- and $y$-axes.

Moving to the right and moving up are regarded as movements in the positive direction. In the coordinate plane, points to the right of $O$ on the $x$-axis and on lines parallel to the $x$-axis and points above $O$ on the $y$-axis and on lines parallel to the $y$-axis are assigned positive values.

Moving to the left and moving down are regarded as movements in the negative direction. In the coordinate plane, points to the left of $O$ on the $x$-axis and on lines parallel to the $x$-axis and points below $O$ on the $y$-axis and on lines parallel to the $y$-axis are assigned negative values.
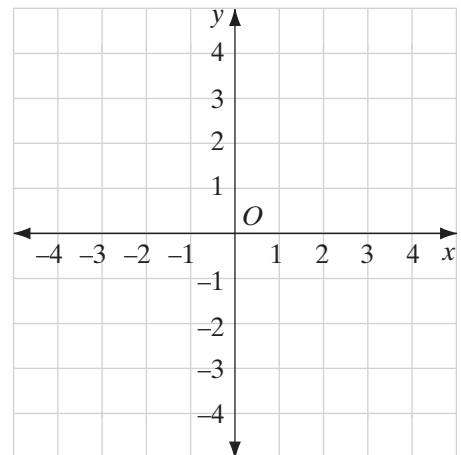
The *x*-axis and the *y*-axis separate the plane into four regions called **quadrants**. These quadrants are numbered I, II, III, and IV in a counterclockwise order, beginning at the upper right, as shown in the accompanying diagram. The points on the axes are not in any quadrant.

## Coordinates of a Point

Every point on the plane can be described by two numbers, called the **coordinates** of the point, usually written as an **ordered pair**. The first number in the pair is called the **x-coordinate** or the **abscissa**. The second number is the **y-coordinate** or the **ordinate**. In general, the coordinates of a point are represented as **(x, y)**.

In the graph at the right, point *A*, which is the **graph of the ordered pair** $(+2, +3)$, lies in quadrant I. Here, *A* lies a distance of 2 units to the right of the origin (in a positive direction along the *x*-axis) and then up a distance of 3 units (in a positive direction parallel to the *y*-axis).



Point *B*, the graph of $(-4, +1)$ in quadrant II, lies a distance of 4 units to the left of the origin (in a negative direction along the *x*-axis) and then up 1 unit (in a positive direction parallel to the *y*-axis).

In quadrant III, every ordered pair $(x, y)$ consists of two negative numbers. For example, *C*, the graph of $(-2, -3)$, lies 2 units to the left of the origin (in the negative direction along the *x*-axis) and then down 3 units (in the negative direction parallel to the *y*-axis).

Point *D*, the graph of $(+3, -5)$ in quadrant IV, lies 3 units to the right of the origin (in a positive direction along the *x*-axis) and then down 5 units (in a negative direction parallel to the *y*-axis).

Point *O*, the origin, has the coordinates $(0, 0)$.
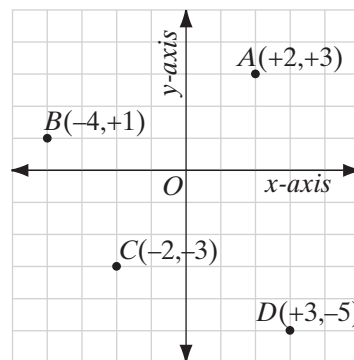
## Locating a Point on the Coordinate Plane

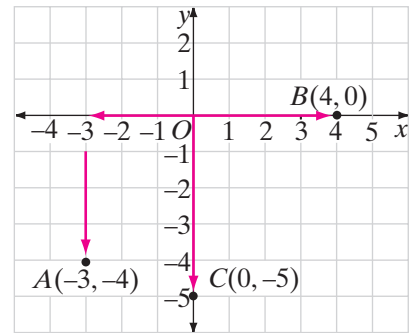An ordered pair of signed numbers uniquely determines the location of a point.

### Procedure

**To find the location of a point on the coordinate plane:**

1. Starting from the origin $O$, move along the $x$-axis the number of units given by the $x$-coordinate. Move to the right if the number is positive or to the left if the number is negative. If the $x$-coordinate is 0, there is no movement along the $x$-axis.

2. Then, from the point on the $x$-axis, move parallel to the $y$-axis the number of units given by the $y$-coordinate. Move up if the number is positive or down if the number is negative. If the $y$-coordinate is 0, there is no movement in the $y$ direction.

To locate the point $A(-3, -4)$, from $O$, move 3 units to the left along the $x$-axis, then 4 units down, parallel to the $y$-axis.

To locate the point $B(4, 0)$, from $O$, move 4 units to the right along the $x$-axis. There is no movement parallel to the $y$-axis.

To locate the point $C(0, -5)$, there is no movement along the $x$-axis. From $O$, move 5 units down along the $y$-axis.

## Finding the Coordinates of a Point on a Plane

The location of a point on the coordinate plane uniquely determines the coordinates of the point.

### Procedure

**To find the coordinates of a point:**

1. From the point, move along a vertical line to the $x$-axis. The number assigned to that point on the $x$-axis is the $x$-coordinate of the point.

2. From the point, move along a horizontal line to the $y$-axis. The number assigned to that point on the $y$-axis is the $y$-coordinate of the point.

To find the coordinates of point $R$, from point $R$, move in the vertical direction to 5 on the $x$-axis and in the horizontal direction to $-6$ on the $y$-axis. The coordinates of $R$ are $(5, -6)$

To find the coordinates of point $S$, from point $S$, move in the vertical direction to $-2$ on the $x$-axis and in the horizontal direction to 4 on the $y$-axis. The coordinates of $S$ are $(-2, 4)$

Point $T$, lies at $-5$ on the $x$-axis. Therefore, the $x$-coordinate is $-5$ and the $y$-coordinate is 0. The coordinates of $T$ are $(-5, 0)$.

Note that if a point lies on the $y$-axis, the $x$-coordinate is 0. If a point lies on the $x$-axis, the $y$-coordinate is 0.
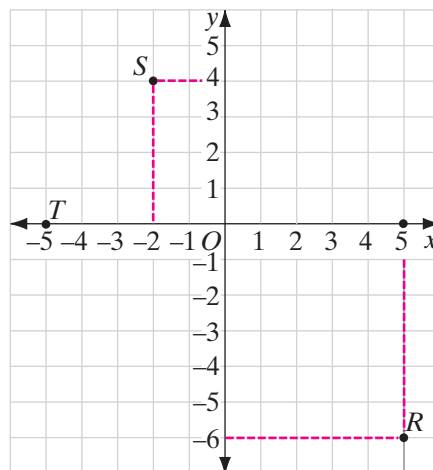
## Graphing Polygons

A **polygon** is a closed figure whose sides are line segments. A **quadrilateral** is a polygon with four sides. The endpoints of the sides are called **vertices**. A quadrilateral can be represented in the coordinate plane by locating its vertices and then drawing the sides, connecting the vertices in order. The graph at the right shows the rectangle $ABCD$. The vertices are $A(3, 2)$, $B(-3, 2)$, $C(-3, -2)$ and $D(3, -2)$. From the graph, note the following:
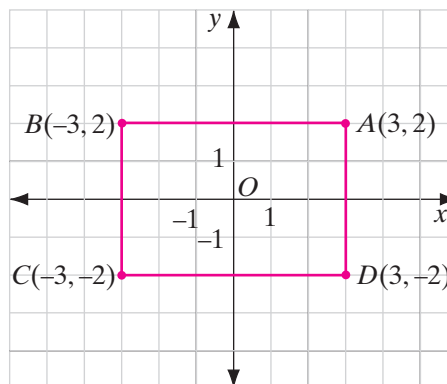
1. Points $A$ and $B$ have the same $y$-coordinate and are on a line parallel to the $x$-axis.

2. Points $C$ and $D$ have the same $y$-coordinate and are on a line parallel to the $x$-axis.

3. Lines parallel to the $x$-axis are parallel to each other.

4. Lines parallel to the $x$-axis are perpendicular to the $y$-axis.

5. Points $B$ and $C$ have the same $x$-coordinate and are on a line parallel to the $y$-axis.

**6.** Points $A$ and $D$ have the same $x$-coordinate and are on a line parallel to the $y$-axis.

**7.** Lines parallel to the $y$-axis are parallel to each other.

**8.** Lines parallel to the $y$-axis are perpendicular to the $x$-axis.

Now, we know that $ABCD$ is a rectangle, because it is a parallelogram with right angles. From the graph, we can find the dimensions of this rectangle. To find the length of the rectangle, we can count the number of units from $A$ to $B$ or from $C$ to $D$. $AB = CD = 6$. Because points on the same horizontal line have the same $y$-coordinate, we can also find $AB$ and $CD$ by subtracting their $x$-coordinates.

$$AB = CD = 3 - (-3) = 3 + 3 = 6$$

To find the width of the rectangle, we can count the number of units from $B$ to $C$ or from $D$ to $A$. $BC = DA = 4$. Because points on the same vertical line have the same $x$-coordinate, we can find $BC$ and $DA$ by subtracting their $y$-coordinates.

$$BC = DA = 2 - (-2) = 2 + 2 = 4$$

### EXAMPLE I

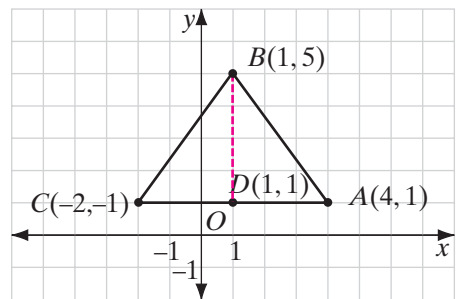Graph the following points: $A(4, 1)$, $B(1, 5)$, $C(-2, 1)$. Then draw $\triangle ABC$ and find its area.

*Solution*   The graph at the right shows $\triangle ABC$.

To find the area of the triangle, we need to know the lengths of the base and of the altitude drawn to that base. The base of $\triangle ABC$ is $\overline{AC}$.

$$AC = 4 - (-2) = 4 + 2 = 6$$

The line segment drawn from $B$ perpendicular to $\overline{AC}$ is the altitude $\overline{BD}$.



$$BD = 5 - 1 = 4$$
$$\text{Area} = \tfrac{1}{2}(AC)(BD)$$
$$= \tfrac{1}{2}(6)(4)$$
$$= 12$$

*Answer*   The area of $\triangle ABC$ is 12 square units.

## EXERCISES

### Writing About Mathematics

**1.** Mark is drawing triangle $ABC$ on the coordinate plane. He locates points $A(-2, -4)$ and $C(5, -4)$. He wants to make $AC = BC$, $\angle C$ a right angle, and point $B$ lie in the first quadrant. What must be the coordinates of point $B$? Explain how you found your answer.

**2.** Phyllis graphed the points $D(3, 0)$, $E(0, 5)$, $F(-2, 0)$, and $G(0, -4)$ on the coordinate plane and joined the points in order. Explain how Phyllis can find the area of this polygon, then find the area.

### Developing Skills

**3.** Write as ordered number pairs the coordinates of points $A$, $B$, $C$, $D$, $E$, $F$, $G$, $H$, and $O$ in the graph.
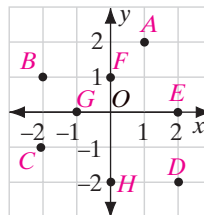


In 4–15, draw a pair of coordinate axes on graph paper and locate the point associated with each ordered number pair. Label each point with its coordinates.

**4.** $(5, 7)$      **5.** $(-3, 2)$      **6.** $(2, -6)$      **7.** $(-4, -5)$

**8.** $(1, 6)$      **9.** $(-8, 5)$      **10.** $(4, -4)$      **11.** $(5, 0)$

**12.** $(-3, 0)$      **13.** $(0, 4)$      **14.** $(0, -6)$      **15.** $(0, 0)$

In 16–20, name the quadrant in which the graph of each point described appears.

**16.** $(5, 7)$      **17.** $(-3, -2)$      **18.** $(-7, 4)$      **19.** $(1, -3)$      **20.** $(|-2|, |-3|)$

**21.** Graph several points on the $x$-axis. What is the value of the $y$-coordinate for every point in the set of points on the $x$-axis?

**22.** Graph several points on the $y$-axis. What is the value of the $x$-coordinate for every point in the set of points on the $y$-axis?

**23.** What are the coordinates of the origin in the coordinate plane?

### Applying Skills

In 24–33: **a.** Graph the points and connect them with straight lines in order, forming a polygon. **b.** Identify the polygon. **c.** Find the area of the polygon.

**24.** $A(1, 1)$, $B(8, 1)$, $C(1, 5)$

**25.** $P(0, 0)$, $Q(5, 0)$, $R(5, 4)$, $S(0, 4)$

**26.** $C(8, -1)$, $A(9, 3)$, $L(4, 3)$, $F(3, -1)$

**27.** $H(-4, 0)$, $O(0, 0)$, $M(0, 4)$, $E(-4, 4)$

**28.** $H(5, -3)$, $E(5, 3)$, $N(-2, 0)$

**29.** $F(5, 1)$, $A(5, 5)$, $R(0, 5)$, $M(-2, 1)$

**30.** $B(-3, -2)$, $A(2, -2)$, $R(2, 2)$, $N(-3, 2)$

**31.** $P(-3, 0)$, $O(0, 0)$, $N(2, 2)$, $D(-1, 2)$

**32.** $R(-4, 2)$, $A(0, 2)$, $M(0, 7)$

**33.** $M(-1, -1)$, $I(3, -1)$, $L(3, 3)$, $K(-1, 3)$

**34.** Graph points $A(1, 1)$, $B(5, 1)$, and $C(5, 4)$. What must be the coordinates of point $D$ if $ABCD$ is a rectangle?

**35.** Graph points $P(-1, -4)$ and $Q(2, -4)$. What are the coordinates of $R$ and $S$ if $PQRS$ is a square? (Two answers are possible.)

**36. a.** Graph points $S(3, 0)$, $T(0, 4)$, $A(-3, 0)$, and $R(0, -4)$, and draw the quadrilateral $STAR$.

    **b.** Find the area of $STAR$ by adding the areas of the triangles into which the axes divide it.

**37. a.** Graph points $P(2, 0)$, $L(1, 1)$, $A(-1, 1)$, $N(-2, 0)$, $E(-1, -1)$, and $T(1, -1)$. Draw $PLANET$, a six-sided polygon called a hexagon.

    **b.** Find the area of $PLANET$. (*Hint:* Use the *x*-axis to separate the hexagon into two parts.).

## CHAPTER SUMMARY

    A **binary operation** in a set assigns to every ordered pair of elements from the set a unique answer from that set. The general form of a binary operation is $a * b = c$, where $a$, $b$, and $c$ are elements of the set and $*$ is the operation symbol. Binary operations exist in arithmetic, in geometry, and in sets.

    Operations in arithmetic include addition, subtraction, multiplication, division, and raising to a power.

    **Powers** are the result of repeated multiplication of the same **factor**, as in $5^3 = 125$. Here, the **base** 5 with an **exponent** of 3 equals $5 \times 5 \times 5$ or 125, the power.

    Numerical expressions are simplified by following a clear **order of operations**:

(1) simplify within parentheses or other grouping symbols;

(2) simplify powers;

(3) multiply and divide from left to right;

(4) add and subtract from left to right.

    Many **properties** are used in operations with real numbers, including:

- **closure** under addition, subtraction, and multiplication;

- **commutative** properties for addition and multiplication, $a * b = b * a$;

- **associative** properties for addition and multiplication, $(a * b) * c = a * (b * c)$;

- **distributive** property of multiplication over addition or subtraction, $a \cdot (b \pm c) = a \cdot b \pm a \cdot c$ and $(a \pm b) \cdot c = a \cdot c \pm b \cdot c$;
- **additive identity (0)**;
- **additive inverses** (opposites), called $-a$ for every element $a$;
- **multiplicative identity (1)**;
- **multiplicative inverses** (reciprocals), $\frac{1}{a}$ for every nonzero element $a$; the nonzero real numbers are closed under division.

Operations with sets include the **intersection** of sets, the **union** of sets, and the **complement** of a set.

Basic operations with signed numbers:

- To add two numbers that have the same sign, find the sum of the absolute values and give this sum the common sign.

- To add two numbers that have different signs, find the difference of the absolute values of the numbers. Give this difference the sign of the number that has the greater absolute value. The sum is 0 if both numbers have the same absolute value.

- To subtract one signed number from another, add the opposite (additive inverse) of the subtrahend to the minuend.

- To multiply two signed numbers, find the product of the absolute values. Write a plus sign before this product when the two numbers have the same sign. Write a minus sign before this product when the two numbers have different signs.

- To divide two signed numbers, find the quotient of the absolute values. Write a plus sign before this quotient when the two numbers have the same sign. Write a minus sign before this quotient when the two numbers have different signs. When 0 is divided by any number, the quotient is 0. Division by 0 is not defined.

The location of a point on a plane is given by an **ordered pair** of numbers that indicate the distance of the point from two reference lines, a horizontal line and a vertical line, called **coordinate axes**. The horizontal line is called the *x*-**axis**, the vertical line is the *y*-**axis**, and their intersection is the **origin**. The pair of numbers that are used to locate a point on the plane are called the **coordinates** of the point. The first number in the pair is called the *x*-**coordinate** or **abscissa**, and the second number is the *y*-**coordinate** or **ordinate**. The coordinates of a point are represented as $(x, y)$.

## VOCABULARY

**2-1** Binary operation • Factor • Prime • Composite • Base • Exponent • Power • Order of operations • Parentheses • Brackets

**2-2** Property • Closure • Commutative property • Associative property •
Distributive property of multiplication over addition and subtraction •
Addition property of zero • Additive identity • Additive inverse
(opposite) • Multiplication property of one • Multiplicative inverse
(reciprocal) • Multiplication property of zero

**2-3** Property of the opposite of a sum

**2-7** Universal set • Intersection • Disjoint sets • Empty set • Union •
Complement

**2-8** *x*-axis • *y*-axis • Coordinate axes • Coordinate plane • Origin • Quadrants
• Coordinates • Ordered pair • *x*-coordinate • Abscissa • *y*-coordinate •
Ordinate • Graph of the ordered pair

**2-9** Polygon • Quadrilateral • Vertices

## REVIEW EXERCISES

In 1–8, simplify each numerical expression.

**1.** $20 - 3 \times 4$

**2.** $(20 - 3) \times (-4)$

**3.** $-8 + 16 \div 4 - 2$

**4.** $(8 - 16) \div (4 - 2)$

**5.** $(0.16)^2$

**6.** $6^2 + 8^2$

**7.** $(6 + 8)^2$

**8.** $7(9 - 7)^3$

In 9–14: **a.** Replace each question mark with a number that makes the sentence
true. **b.** Name the property illustrated in each sentence that is formed when the
replacement is made.

**9.** $8 + (2 + 9) = 8 + (9 + ?)$

**10.** $8 + (2 + 9) = (8 + ?) + 9$

**11.** $3(?) = 3$

**12.** $3(?) = 0$

**13.** $5(7 + 4) = 5(7) + ?(4)$

**14.** $5(7 + 4) = (7 + 4)?$

In 15–24, the universe $U = \{1, 2, 3, 4, 5, 6\}$, set $A = \{1, 2, 4, 5\}$, and set $B = \{2, 4, 6\}$.
In each case, perform the given operation and list the element(s) of the result-
ing set.

**15.** $A \cap B$

**16.** $A \cup B$

**17.** $\overline{A}$

**18.** $\overline{A} \cap B$

**19.** $A \cap \overline{A}$

**20.** $A \cup \overline{A}$

**21.** $A \cap \overline{B}$

**22.** $A \cup \overline{B}$

**23.** $\overline{A} \cap \overline{B}$

**24.** $\overline{A} \cup \overline{B}$

In 25–32, find each sum or difference.

**25.** $|6 - 6|$

**26.** $|6| + |-6|$

**27.** $|-3.2| - |-4.5|$

**28.** $|4| - |-5|$

**29.** $|-23| + |0|$

**30.** $|-54 + 52|$

**31.** $|100| + |-25|$

**32.** $|0 - 7|$

In 33–40, to each property named in Column I, match the correct application of the property found in Column II.

| *Column I* | *Column II* |
|---|---|
| **33.** Associative property of multiplication | **a.** $3 + 4 = 4 + 3$ |
| **34.** Associative property of addition | **b.** $3 \times 1 = 3$ |
| **35.** Commutative property of addition | **c.** $0 \times 4 = 0$ |
| **36.** Commutative property of multiplication | **d.** $3 + 0 = 3$ |
| **37.** Identity element of multiplication | **e.** $3 \times 4 = 4 \times 3$ |
| **38.** Identity element of addition | **f.** $3(4 + 5) = 3(4) + 3(5)$ |
| **39.** Distributive property | **g.** $3(4 \times 5) = (3 \times 4)5$ |
| **40.** Multiplication property of zero | **h.** $(3 + 4) + 5 = 3 + (4 + 5)$ |

**41. a.** Find the number on the odometer of Jesse's car described in the chapter opener on page 37.

   **b.** What would the number be if it were an odd number?

**42. a.** Rectangle $ABCD$ is drawn with two sides parallel to the $x$-axis. The coordinates of vertex $A$ are $(-2, -4)$ and the coordinates of $C$ are $(3, 5)$. Find the coordinates of vertices $B$ and $D$.

   **b.** What is the area of rectangle $ABCD$?

**43.** Maurice answered all of the 60 questions on a multiple-choice test. The test was scored by using the formula $S = R - \frac{W}{4}$ where $S$ is the score on the test, $R$ is the number right, and $W$ is the number wrong.

   **a.** What is the lowest possible score?

   **b.** How many answers did Maurice get right if his score was $-5$?

   **c.** Is it possible for a person who answers all of the questions to get a score of $-4$? Explain why not or find the numbers of right and wrong answers needed to get this score.

   **d.** Is it possible for a person who does not answer all of the questions to get a score of $-4$? Explain why not or find the numbers of right and wrong answers needed to get this score.

**44.** Solve the following problem using signed numbers.

   Doug, the team's replacement quarterback, started out on his team's 30-yard line. On the first play, one of his linemen was offsides for a loss of 5 yards. On the next play, Doug gave the ball to the runningback who made a gain of 8 yards. He then made a 17-yard pass. Then Doug was tackled, for a loss of 3 yards. Where was Doug on the field after he was tackled?

### Exploration

In this activity, you will derive a rule to determine if a number is divisible by 3. We will start our exploration with the number 23,568. For steps 1–4, fill in the blanks with a digit that will make the equality true.

**STEP 1.** Write the number as a sum of powers of ten.

$$23{,}568 = 20{,}000 + 3{,}000 + 500 + 60 + 8$$
$$= \square \times 10{,}000 + \square \times 1{,}000 + \square \times 100 + \square \times 10 + \square$$

**STEP 2.** Rewrite the powers of ten as a multiple of 9, plus 1.

$$\square \times 10{,}000 \quad + \square \times 1{,}000 \quad + \square \times 100 \quad + \square \times 10 \quad + \square$$
$$= \square \times (9{,}999 + 1) + \square \times (999 + 1) + \square \times (99 + 1) + \square \times (9 + 1) + \square$$

**STEP 3.** Use the distributive property to expand the product terms. Do not multiply out products involving the multiples of 9.

$$\square \times (9{,}999 + 1) + \square \times (999 + 1) + \square \times (99 + 1) + \square \times (9 + 1) + \square$$
$$= (\square \times 9{,}999 + \square) + (\square \times 999 + \square) + (\square \times 99 + \square) + (\square \times 9 + \square) + \square$$

**STEP 4.** Group the products involving the multiples of 9 first and then the remaining digit terms.

$$(\square \times 9{,}999 + \square) + (\square \times 999 + \square) + (\square \times 99 + \square) + (\square \times 9 + \square) + \square$$
$$= (\square \times 9{,}999 + \square \times 999 + \square \times 99 + \square \times 9) + (\square + \square + \square + \square + \square)$$

**STEP 5.** Compare the expression involving the digit terms with the original number. What do they have in common?

**STEP 6.** The expression involving the multiples of 9 is divisible by 3. Why?

**STEP 7.** If the expression involving the digit terms is divisible by 3, will the entire expression be divisible by 3? Explain.

**STEP 8.** Based on steps 1–7 write the rule to determine if a number is divisible by 3.

## CUMULATIVE REVIEW                              CHAPTERS 1–2

### Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

**1.** Which number is not an integer?

(1) 7          (2) $-2$          (3) 0.2          (4) $\sqrt{9}$

**2.** Which inequality is false?

(1) $4 > 3$        (2) $3 \geq 3$        (3) $-4 > -3$        (4) $+3 > -3$

**3.** What is the opposite of $-4$?

(1) $-\frac{1}{4}$        (2) $\frac{1}{4}$        (3) $+4$        (4) $-4$

**4.** Which of the following numbers has the greatest value?

(1) $\frac{5}{7}$        (2) $0.7$        (3) $\frac{\pi}{5}$        (4) $\frac{25}{36}$

**5.** Under which operation is the set of integers *not* closed?

(1) Addition      (2) Subtraction      (3) Multiplication      (4) Division

**6.** Which of the following numbers has exactly three factors?

(1) $1$        (2) $2$        (3) $9$        (4) $15$

**7.** The graph of the ordered pair $(-3, 5)$ is in which quadrant?

(1) I        (2) II        (3) III        (4) IV

**8.** Put the following numbers in order, starting with the smallest:

$-\frac{1}{3}, -\frac{1}{5}, -\frac{2}{7}$

(1) $-\frac{1}{3} < -\frac{2}{7} < -\frac{1}{5}$               (3) $-\frac{1}{5} < -\frac{1}{3} < -\frac{2}{7}$

(2) $-\frac{1}{3} < -\frac{1}{5} < -\frac{2}{7}$               (4) $-\frac{1}{5} < -\frac{2}{7} < -\frac{1}{3}$

**9.** When rounded to the nearest hundredth, $\sqrt{3}$ is approximately equal to

(1) $1.732$        (2) $1.730$        (3) $1.73$        (4) $1.74$

**10.** For which value of $t$ is $\frac{1}{t} > t > t^2$?

(1) $1$        (2) $2$        (3) $0$        (4) $\frac{1}{2}$

## Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

**11.** Mrs. Ling spends more than $4.90 and less than $5.00 for meat for tonight's dinner. Write the set of all possible amounts that she could have paid for the meat. Is this a finite or an infinite set? Explain.

**12.** In a basketball league, 100 students play on 8 teams. Each team has at least 12 players. What is the largest possible number of players on any one team?

## Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

**13.** A teacher wrote the sequence 2, 4, 6, . . . and asked the class what the next number could be. Three students each gave a different answer. The teacher said that each of the answers was correct.

**a.** Josie said 8. Explain the rule that she used.

**b.** Emil said 10. Explain the rule that he used.

**c.** Ross said 12. Explain the rule that he used.

**14.** Evaluate the following expression without using a calculator. Show each step in your computation.

$$4(-7 + 3) - 8 \div (-2 + 6)^2$$

## Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

**15.** The vertices of triangle $ABC$ are $A(-2, -3)$, $B(5, -3)$, and $C(0, 4)$. Draw triangle $ABC$ on the coordinate plane and find its area.

**16.** A survey to which 250 persons responded found that 140 persons said that they watch the news on TV at 6 o'clock, 120 persons said that they watch the news on TV at 11 o'clock and 40 persons said that they do not watch the news on TV at any time.

**a.** How many persons from this group watch the news both at 6 and at 11?

**b.** How many persons from this group watch the news at 6 but not at 11?

**c.** How many persons from this group watch the news at 11 but not at 6?